UNIT 1

Lines and Angles

Lines and angles are all around us and can be used to model and describe real-world situations. In this unit, you will learn about lines, planes, and angles and how they can be used to prove theorems.

WebQuest Internet Project

When Is Weather Normal?

Source: *USA TODAY,* October 8, 2000

"Climate normals are a useful way to describe the average weather of a location. Several statistical measures are computed as part of the normals, including measures of central tendency, such as mean or median, of dispersion or how spread out the values are, such as the standard deviation or inter-quartile range, and of frequency or probability of occurrence." In this project, you will explore how latitude, longitude, and *degree distance* relate to differences in temperature for pairs of U.S. cities.

Log on to www.geometryonline.com/webquest. Begin your WebQuest by reading the Task.

Continue working on your WebQuest as you study Unit 1.

Lesson	1-3	2-1	3-5
Page	23	65	155

USA TODAY Snapshots®

Coldest cities in the USA

City	Mean temperature
International Falls, Minn.	**36.4**
Duluth, Minn.	**38.2**
Caribou, Maine	**38.9**
Marquette, Mich.	**39.2**
Sault Ste. Marie, Mich.	**39.7**
Williston, N.D.	**40.1**
Fargo, N.D.	**40.5**
Alamosa, Colo.	**41.2**
Bismarck, N.D.	**41.3**
St. Cloud, Minn.	**41.4**

Source: Planet101.com By Lori Joseph and Keith Simmons, USA TODAY

Chapter 1

Points, Lines, Planes, and Angles

What You'll Learn

- **Lesson 1-1** Identify and model points, lines, and planes.
- **Lesson 1-2** Measure segments and determine accuracy of measurements.
- **Lesson 1-3** Calculate the distance between points and find the midpoint of a segment.
- **Lessons 1-4 and 1-5** Measure and classify angles and identify angle relationships.
- **Lesson 1-6** Identify polygons and find their perimeters.

Key Vocabulary

- **line segment** (p. 13)
- **congruent** (p. 15)
- **segment bisector** (p. 24)
- **angle bisector** (p. 32)
- **perpendicular** (p. 40)

Why It's Important

Points, lines, and planes are the basic building blocks used in geometry. They can be used to describe real-world objects. For example, a kite can model lines, angles, and planes in two and three dimensions. *You will explore the angles formed by the structure of a kite in Lesson 1-2.*

Roy Morsch/CORBIS

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 1.

For Lesson 1-1 — Graph Points

Graph and label each point in the coordinate plane. *(For review, see pages 728 and 729.)*

1. $A(3, -2)$ **2.** $B(4, 0)$ **3.** $C(-4, -4)$ **4.** $D(-1, 2)$

For Lesson 1-2 — Add and Subtract Fractions

Find each sum or difference.

5. $\frac{3}{4} + \frac{3}{8}$ **6.** $2\frac{5}{16} + 5\frac{1}{8}$ **7.** $\frac{7}{8} - \frac{9}{16}$ **8.** $11\frac{1}{2} - 9\frac{7}{16}$

For Lessons 1-3 through 1-5 — Operations With Integers

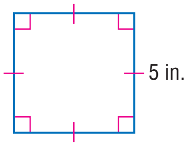
Evaluate each expression. *(For review, see pages 734 and 735.)*

9. $2 - 17$ **10.** $23 - (-14)$ **11.** $[-7 - (-2)]^2$ **12.** $9^2 + 13^2$

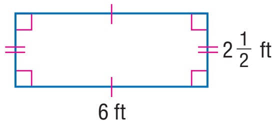
For Lesson 1-6 — Find Perimeter

Find the perimeter of each figure. *(For review, see pages 732 and 733.)*

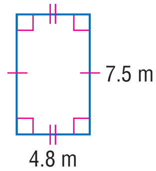
13.



14.



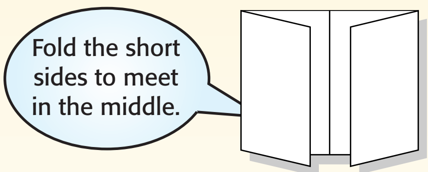
15.



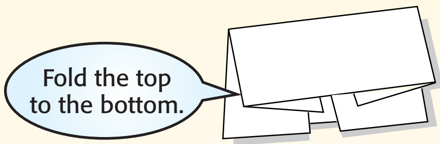
FOLDABLES™ Study Organizer

Lines and Angles Make this Foldable to collect examples of and notes about lines and angles. Begin with a sheet of paper.

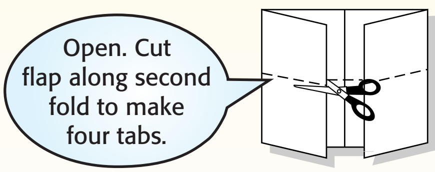
Step 1 Fold



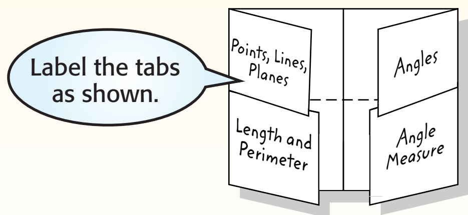
Step 2 Fold Again



Step 3 Cut



Step 4 Label



Reading and Writing As you read and study the chapter, record examples and notes from each lesson under the appropriate tab.

1-1 Points, Lines, and Planes

What You'll Learn

- Identify and model points, lines, and planes.
- Identify collinear and coplanar points and intersecting lines and planes in space.

Why do chairs sometimes wobble?

Have you ever noticed that a four-legged chair sometimes wobbles, but a three-legged stool never wobbles? This is an example of points and how they lie in a plane. All geometric shapes are made of points. In this book, you will learn about those shapes and their characteristics.

Vocabulary

- point
- line
- collinear
- plane
- coplanar
- undefined term
- space
- locus

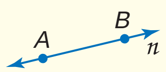
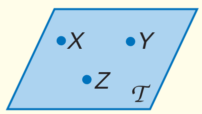
NAME POINTS, LINES, AND PLANES You are familiar with the terms *plane*, *line*, and *point* from algebra. You graph on a coordinate *plane*, and ordered pairs represent *points* on *lines*. In geometry, these terms have similar meanings.

Unlike objects in the real world that model these shapes, points, lines, and planes do not have any actual size.

- A **point** is simply a location.
- A **line** is made up of points and has no thickness or width. Points on the same line are said to be **collinear**.
- A **plane** is a flat surface made up of points. Points that lie on the same plane are said to be **coplanar**. A plane has no depth and extends infinitely in all directions.

Points are often used to name lines and planes.

Key Concept: Points, Lines, and Planes

	Point	Line	Plane
Model:			
Drawn:	as a dot	with an arrowhead at each end	as a shaded, slanted 4-sided figure
Named by:	a capital letter	the letters representing two points on the line or a lowercase script letter	a capital script letter or by the letters naming three noncollinear points
Facts	A point has neither shape nor size.	There is exactly one line through any two points.	There is exactly one plane through any three noncollinear points.
Words/ Symbols	point P	line n, line AB or $\overleftrightarrow{AB}$, line BA or $\overleftrightarrow{BA}$	plane $\mathcal{T}$, plane XYZ, plane XZY, plane YXZ, plane YZX, plane ZXY, plane ZYX

Study Tip

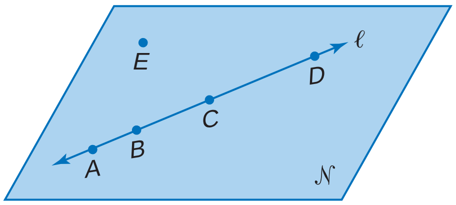
Reading Math

The word *noncollinear* means not collinear or not lying on the same line. Likewise, *noncoplanar* means not lying in the same plane.

Notice that the letters can be in any order when naming lines or planes.

Example 1 Name Lines and Planes

Use the figure to name each of the following.



a. a line containing point *A*

The line can be named as line ℓ.

There are four points on the line. Any two of the points can be used to name the line.

$\overleftrightarrow{AB}$ $\overleftrightarrow{BA}$ $\overleftrightarrow{AC}$ $\overleftrightarrow{CA}$ $\overleftrightarrow{AD}$ $\overleftrightarrow{DA}$

$\overleftrightarrow{BC}$ $\overleftrightarrow{CB}$ $\overleftrightarrow{BD}$ $\overleftrightarrow{DB}$ $\overleftrightarrow{CD}$ $\overleftrightarrow{DC}$

b. a plane containing point *C*

The plane can be named as plane $\mathcal{N}$.

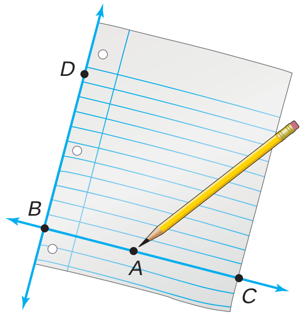
You can also use the letters of any three *noncollinear* points to name the plane.

plane *ABE* plane *ACE* plane *ADE* plane *BCE* plane *BDE* plane *CDE*

The letters of each of these names can be reordered to create other acceptable names for this plane. For example, *ABE* can also be written as *AEB*, *BEA*, *BAE*, *EBA*, and *EAB*. In all, there are 36 different three-letter names for this plane.

Example 2 Model Points, Lines, and Planes

VISUALIZATION **Name the geometric shapes modeled by the picture.**



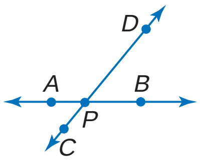
The pencil point models point *A*.

The blue rule on the paper models line *BC*.

The edge of the paper models line *BD*.

The sheet of paper models plane *ADC*.

In geometry, the terms *point*, *line*, and *plane* are considered **undefined terms** because they have only been explained using examples and descriptions. Even though they are undefined, these terms can still be used to define other geometric terms and properties. For example, two lines intersect in a point. In the figure at the right, point *P* represents the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. Lines can intersect planes, and planes can intersect each other.



Study Tip

Naming Points
Points on the coordinate plane are named using *ordered pairs*. Point *G* can be named as *G*(−1, −3).

Example 3 Draw Geometric Figures

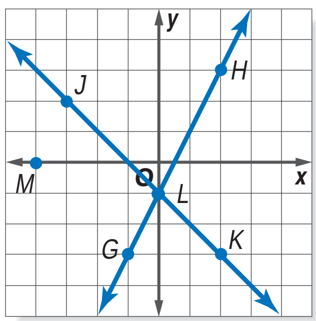
Draw and label a figure for each relationship.

a. ALGEBRA **Lines *GH* and *JK* intersect at *L* for *G*(−1, −3), *H*(2, 3), *J*(−3, 2), and *K*(2, −3) on a coordinate plane. Point *M* is coplanar with these points, but not collinear with $\overleftrightarrow{GH}$ or $\overleftrightarrow{JK}$.**

Graph each point and draw $\overleftrightarrow{GH}$ and $\overleftrightarrow{JK}$.

Label the intersection point as *L*.

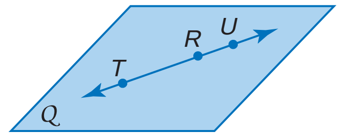
There are an infinite number of points that are coplanar with *G*, *H*, *J*, *K*, and *L*, but are not collinear with $\overleftrightarrow{GH}$ or $\overleftrightarrow{JK}$. In the graph, one such point is *M*(−4, 0).



b. $\overleftrightarrow{TU}$ lies in plane *Q* and contains point *R*.

Draw a surface to represent plane *Q* and label it.
Draw a line anywhere on the plane.
Draw dots on the line for points *T* and *U*.
Since $\overleftrightarrow{TU}$ contains *R*, point *R* lies on $\overleftrightarrow{TU}$.
Draw a dot on $\overleftrightarrow{TU}$ and label it *R*.

The locations of points *T*, *R*, and *U* are totally arbitrary.



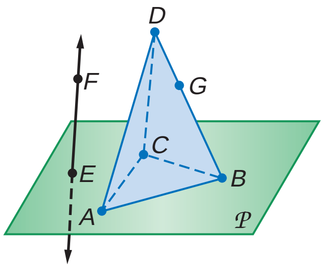
POINTS, LINES, AND PLANES IN SPACE

Space is a boundless, three-dimensional set of all points. Space can contain lines and planes.

Study Tip

Three-Dimensional Drawings
Because it is impossible to show space or an entire plane in a figure, edged shapes with different shades of color are used to represent planes. If the lines are hidden from view, the lines or segments are shown as dashed lines or segments.

Example 4 Interpret Drawings



a. How many planes appear in this figure?
There are four planes: plane *𝒫*, plane *ADB*, plane *BCD*, plane *ACD*

b. Name three points that are collinear.
Points *D*, *B*, and *G* are collinear.

c. Are points *G*, *A*, *B*, and *E* coplanar? Explain.
Points *A*, *B*, and *E* lie in plane *𝒫*, but point *G* does not lie in plane *𝒫*. Thus, they are not coplanar. Points *A*, *G*, and *B* lie in a plane, but point *E* does not lie in plane *AGB*.

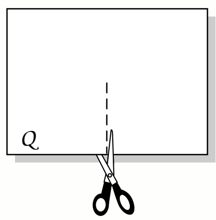
d. At what point do $\overleftrightarrow{EF}$ and $\overleftrightarrow{AB}$ intersect?
$\overleftrightarrow{EF}$ and $\overleftrightarrow{AB}$ do not intersect. $\overleftrightarrow{AB}$ lies in plane *𝒫*, but only point *E* of $\overleftrightarrow{EP}$ lies in *𝒫*.

Sometimes it is difficult to identify collinear or coplanar points in space unless you understand what a drawing represents. In geometry, a model is often helpful in understanding what a drawing is portraying.

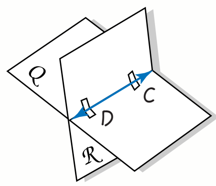
Geometry Activity

Modeling Intersecting Planes

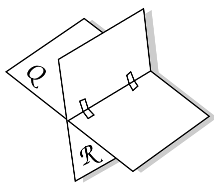
- Label one index card as *Q* and another as *ℛ*.
- Hold the two index cards together and cut a slit halfway through both cards.



- Where the two cards meet models a line. Draw the line and label two points, *C* and *D*, on the line.



- Hold the cards so that the slits meet and insert one card into the slit of the other. Use tape to hold the cards together.



Analyze

1. Draw a point *F* on your model so that it lies in *Q* but not in *ℛ*. Can *F* lie on $\overline{DC}$?
2. Draw point *G* so that it lies in *ℛ*, but not in *Q*. Can *G* lie on $\overline{DC}$?
3. If point *H* lies in both *Q* and *ℛ*, where would it lie? Draw point *H* on your model.
4. Draw a sketch of your model on paper. Label all points, lines, and planes appropriately.

Check for Understanding

Concept Check

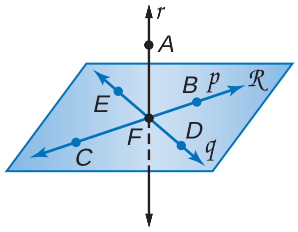
1. **Name** three undefined terms from this lesson.
2. **OPEN ENDED** Fold a sheet of paper. Open the paper and fold it again in a different way. Open the paper and label the geometric figures you observe. Describe the figures.
3. **FIND THE ERROR** Raymond and Micha were looking for patterns to determine how many ways there are to name a plane given a certain number of points.

Raymond	Micha
If there are 4 points, then there are 4 · 3 · 2 ways to name the plane.	If there are 5 noncollinear points, then there are 5 · 4 · 3 ways to name the plane.

Who is correct? Explain your reasoning.

Guided Practice

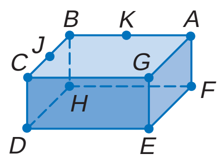
4. Use the figure at the right to name a line containing point *B* and a plane containing points *D* and *C*.



Draw and label a figure for each relationship.

5. A line in a coordinate plane contains *X*(3, −1), *Y*(−3, −4), and *Z*(−1, −3) and a point *W* that does not lie on $\overleftrightarrow{XY}$.
6. Plane *Q* contains lines *r* and *s* that intersect in *P*.

Refer to the figure.



7. How many planes are shown in the figure?
8. Name three points that are collinear.
9. Are points *A*, *C*, *D*, and *J* coplanar? Explain.

Application

VISUALIZATION **Name the geometric term modeled by each object.**

10. 

11. a pixel on a computer screen
12. a ceiling

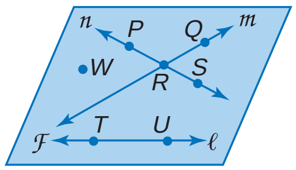
Practice and Apply

Homework Help

For Exercises	See Examples
13–18	1
21–28	3
30–37	4
38–46	2

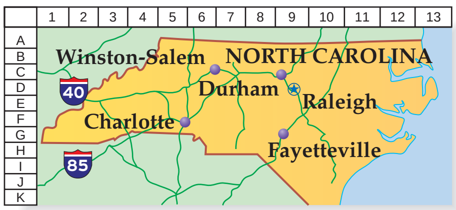
Extra Practice
See page 754.

Refer to the figure.



13. Name a line that contains point *P*.
14. Name the plane containing lines *n* and *m*.
15. Name the intersection of lines *n* and *m*.
16. Name a point not contained in lines ℓ, *m*, or *n*.
17. What is another name for line *n*?
18. Does line ℓ intersect line *m* or line *n*? Explain.

MAPS **For Exercises 19 and 20, refer to the map, and use the following information.** A map represents a plane. Points on this plane are named using a letter/number combination.



19. Name the point where Raleigh is located.
20. What city is located at (F, 5)?

Draw and label a figure for each relationship.

21. Line AB intersects plane Q at W.

22. Point T lies on $\overleftrightarrow{WR}$.

23. Points $Z(4, 2)$, $R(-4, 2)$, and S are collinear, but points Q, Z, R, and S are not.

24. The coordinates for points C and R are $(-1, 4)$ and $(6, 4)$, respectively. $\overleftrightarrow{RS}$ and $\overleftrightarrow{CD}$ intersect at $P(3, 2)$.

25. Lines a, b, and c are coplanar, but do not intersect.

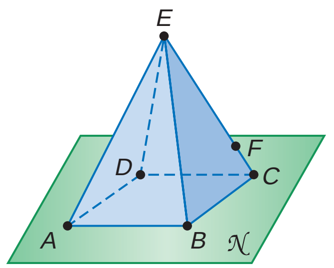
26. Lines a, b, and c are coplanar and meet at point F.

27. Point C and line r lie in $\mathcal{M}$. Line r intersects line s at D. Point C, line r, and line s are not coplanar.

28. Planes $\mathcal{A}$ and $\mathcal{B}$ intersect in line s. Plane $\mathcal{C}$ intersects $\mathcal{A}$ and $\mathcal{B}$, but does not contain s.

29. **ALGEBRA** Name at least four ordered pairs for which the sum of coordinates is -2. Graph them and describe the graph.

Refer to the figure.



30. How many planes are shown in the figure?

31. How many planes contain points B, C, and E?

32. Name three collinear points.

33. Where could you add point G on plane $\mathcal{N}$ so that A, B, and G would be collinear?

34. Name a point that is not coplanar with A, B, and C.

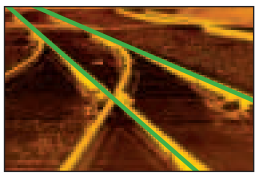
35. Name four points that are coplanar.

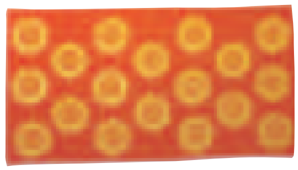
36. Name an example that shows that three points are always coplanar, but four points are not always coplanar.

37. Name the intersection of plane $\mathcal{N}$ and the plane that contains points A, E, and C.

VISUALIZATION **Name the geometric term(s) modeled by each object.**

38. 

39. 

40. 

41. a table cloth

42. a partially-opened newspaper

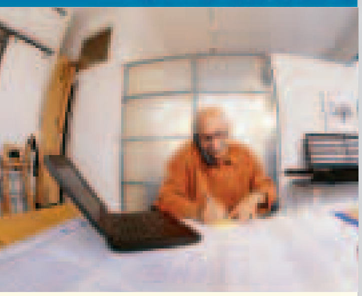
43. a star in the sky

44. woven threads in a piece of cloth

45. a knot in a string

46. satellite dish signal

Career Choices

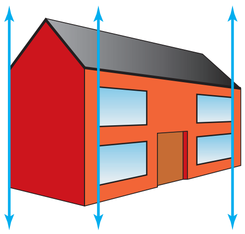


Engineering Technician

Engineering technicians or drafters use perspective to create drawings used in construction, and manufacturing. Technicians must have knowledge of math, science, and engineering.

Online Research
For information about a career as an engineering technician, visit: www.geometryonline.com/careers

ONE-POINT PERSPECTIVE **One-point perspective drawings use lines to convey depth in a picture. Lines representing horizontal lines in the real object can be extended to meet at a single point called the *vanishing point*.**



47. Trace the figure at the right. Draw all of the vertical lines. Several are already drawn for you.

48. Draw and extend the horizontal lines to locate the vanishing point and label it.

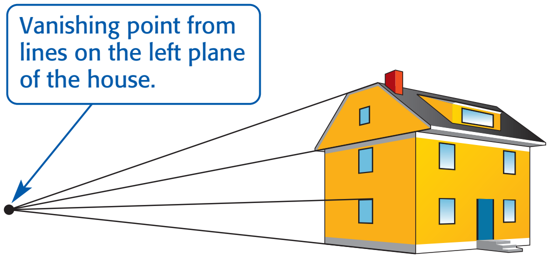
49. Draw a one-point perspective of your classroom or a room in your house.

(l)Daniel Aubry/CORBIS, (cl)Aaron Haupt, (cr)Donovan Reese/PhotoDisc, (r)Laura Sifferlin

50. **RESEARCH** Use the Internet or other research resources to investigate one-point perspective drawings in which the vanishing point is in the center of the picture. How do they differ from the drawing for Exercises 47–49?

TWO-POINT PERSPECTIVE **Two-point perspective drawings also use lines to convey depth, but two sets of lines can be drawn to meet at two vanishing points.**

51. Trace the outline of the house. Draw all of the vertical lines.



52. Draw and extend the lines on your sketch representing horizontal lines in the real house to identify the vanishing point on the right plane in this figure.

53. Which types of lines seem unaffected by any type of perspective drawing?

54. **CRITICAL THINKING** Describe a real-life example of three lines in space that do not intersect each other and no two lines lie in the same plane.

55. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why do chairs sometimes wobble?

Include the following in your answer:

- an explanation of how the chair legs relate to points in a plane, and
- how many legs would create a chair that does not wobble.

Standardized Test Practice

56. Four lines are coplanar. What is the greatest number of intersection points that can exist?

(A) 4 (B) 5 (C) 6 (D) 7

57. **ALGEBRA** If $2 + x = 2 - x$, then $x = ?$

(A) -1 (B) 0 (C) 1 (D) 2

Extending the Lesson

Another way to describe a group of points is called a locus. A locus is a set of points that satisfy a particular condition.

58. Find five points that satisfy the equation $4 - x = y$. Graph them on a coordinate plane and describe the geometric figure they suggest.

59. Find ten points that satisfy the inequality $y > -2x + 1$. Graph them on a coordinate plane and describe the geometric figure they suggest.

Getting Ready for the Next Lesson

BASIC SKILL **Replace each ● with >, <, or = to make a true statement.**

60. $\frac{1}{2}$ in. ● $\frac{3}{8}$ in.
61. $\frac{4}{16}$ in. ● $\frac{1}{4}$ in.
62. $\frac{4}{5}$ in. ● $\frac{6}{10}$ in.
63. 10 mm ● 1 cm
64. 2.5 cm ● 28 mm
65. 0.025 cm ● 25 mm

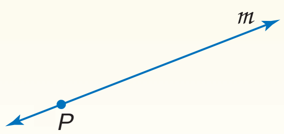
www.geometryonline.com/self_check_quiz

Reading Mathematics

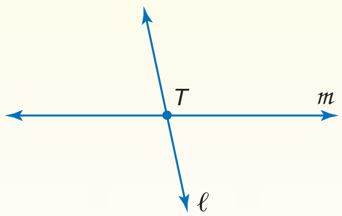
Describing What You See

Figures play an important role in understanding geometric concepts. It is helpful to know what words and phrases can be used to describe figures. Likewise, it is important to know how to read a geometric description and be able to draw the figure it describes.

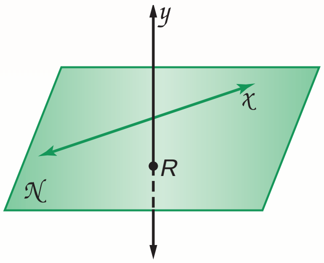
The figures and descriptions below help you visualize and write about points, lines, and planes.



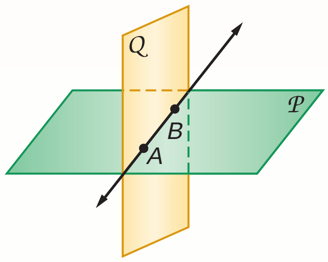
Point P is on m.
Line m contains P.
Line m passes through P.



Lines ℓ and m intersect in T.
Point T is the intersection of ℓ and m.
Point T is on m. Point T is on ℓ.



Line x and point R are in $\mathcal{N}$.
Point R lies in $\mathcal{N}$.
Plane $\mathcal{N}$ contains R and x.
Line y intersects $\mathcal{N}$ at R.
Point R is the intersection of y with $\mathcal{N}$.
Lines y and x do not intersect.

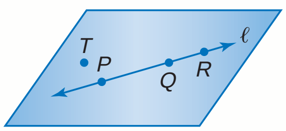


$\overleftrightarrow{AB}$ is in $\mathcal{P}$ and $\mathcal{Q}$.
Points A and B lie in both $\mathcal{P}$ and $\mathcal{Q}$.
Planes $\mathcal{P}$ and $\mathcal{Q}$ both contain $\overleftrightarrow{AB}$.
Planes $\mathcal{P}$ and $\mathcal{Q}$ intersect in $\overleftrightarrow{AB}$.
$\overleftrightarrow{AB}$ is the intersection of $\mathcal{P}$ and $\mathcal{Q}$.

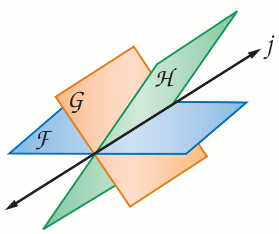
Reading to Learn

Write a description for each figure.

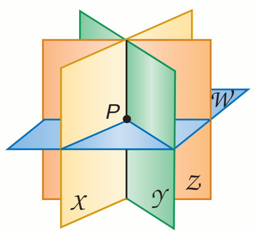
1.



2.



3.



4. Draw and label a figure for the statement *Planes $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ do not intersect.*

1-2 Linear Measure and Precision

What You'll Learn

- Measure segments and determine accuracy of measurement.
- Compute with measures.

Why are units of measure important?

When you look at the sign, you probably assume that the unit of measure is miles. However, if you were in France, this would be 17 kilometers, which is a shorter distance than 17 miles. Units of measure give us points of reference when evaluating the sizes of objects.

Vocabulary

- line segment
- precision
- betweenness of points
- between
- congruent
- construction
- relative error

MEASURE LINE SEGMENTS Unlike a line, a **line segment**, or *segment*, can be measured because it has two endpoints. A segment with endpoints A and B can be named as $\overline{AB}$ or $\overline{BA}$. The length or measure of $\overline{AB}$ is written as AB. The length of a segment is only as precise as the smallest unit on the measuring device.

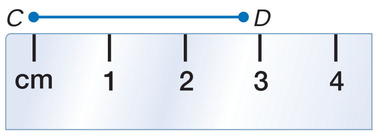
Study Tip

Using a Ruler
On a ruler, the smallest unit is frequently labeled as cm, mm, or 16th of an inch. The zero point on a ruler may not be clearly marked. For some rulers, zero is the left edge of the ruler. On others, it may be a line farther in on the scale. If it is not clear where zero is, align the endpoint on 1 and subtract 1 from the measurement at the other endpoint.

Example 1 Length in Metric Units

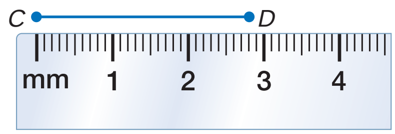
Find the length of $\overline{CD}$ using each ruler.

a.



The ruler is marked in centimeters. Point D is closer to the 3-centimeter mark than to 2 centimeters. Thus, $\overline{CD}$ is about 3 centimeters long.

b.

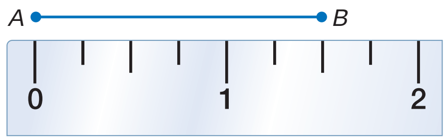


The long marks are centimeters, and the shorter marks are millimeters. There are 10 millimeters for each centimeter. Thus, $\overline{CD}$ is about 28 millimeters long.

Example 2 Length in Customary Units

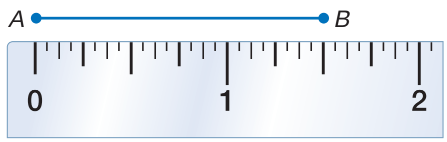
Find the length of $\overline{AB}$ using each ruler.

a.



Each inch is divided into fourths. The long marks are half-inch increments. Point B is closer to the $1\frac{2}{4}$-inch mark. Thus, $\overline{AB}$ is about $1\frac{2}{4}$ or $1\frac{1}{2}$ inches long.

b.



Each inch is divided into sixteenths. Point B is closer to the $1\frac{8}{16}$-inch mark. Thus, $\overline{AB}$ is about $1\frac{8}{16}$ or $1\frac{1}{2}$ inches long.

Study Tip

Units of Measure

A measurement of 38.0 centimeters on a ruler with millimeter marks means a measurement of 380 millimeters. So the actual measurement is between 379.5 millimeters and 380.5 millimeters, not 37.5 centimeters and 38.5 centimeters. For a measurement of $2\frac{1}{2}$ inches, you can only assume that the ruler has half-inch increments. A measurement of $2\frac{2}{4}$ inches indicates that the ruler has one-fourth inch increments.

Study Tip

Comparing Measures

Because measures are real numbers, you can compare measures. If X, Y, and Z are collinear in that order, then one of these statements is true. $XY = YZ$, $XY > YZ$, or $XY < YZ$.

The **precision** of any measurement depends on the smallest unit available on the measuring tool. The measurement should be precise to within 0.5 unit of measure. For example, in part **a** of Example 1, 3 centimeters means that the actual length is no less than 2.5 centimeters, but no more than 3.5 centimeters.

Measurements of 28 centimeters and 28.0 centimeters indicate different precision in measurement. A measurement of 28 centimeters means that the ruler is divided into centimeters. However, a measurement of 28.0 centimeters indicates that the ruler is divided into millimeters.

Example 3 Precision

Find the precision for each measurement. Explain its meaning.

a. 5 millimeters

The measurement is precise to within 0.5 millimeter. So, a measurement of 5 millimeters could be 4.5 to 5.5 millimeters.

b. $8\frac{1}{2}$ inches

The measuring tool is divided into $\frac{1}{2}$-inch increments. Thus, the measurement is precise to within $\frac{1}{2}\left(\frac{1}{2}\right)$ or $\frac{1}{4}$ inch. Therefore, the measurement could be between $8\frac{1}{4}$ inches and $8\frac{3}{4}$ inches.

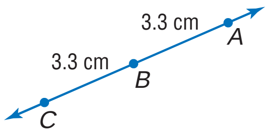
CALCULATE MEASURES

Measures are real numbers, so all arithmetic operations can be used with them. You know that the whole usually equals the sum of its parts. That is also true of line segments in geometry.

Recall that for any two real numbers a and b, there is a real number n between a and b such that $a < n < b$. This relationship also applies to points on a line and is called **betweenness of points**. Point M is **between** points P and Q if and only if P, Q, and M are collinear and $PM + MQ = PQ$.

Example 4 Find Measurements

a. Find AC.



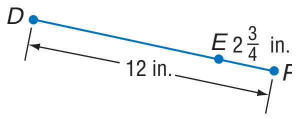
AC is the measure of $\overline{AC}$.

Point B is between A and C. AC can be found by adding AB and BC.

$AB + BC = AC$	Sum of parts = whole
$3.3 + 3.3 = AC$	Substitution
$6.6 = AC$	Add.

So, $\overline{AC}$ is 6.6 centimeters long.

b. Find DE.



DE is the measure of $\overline{DE}$.

$DE + EF = DF$	Sum of parts = whole
$DE + 2\frac{3}{4} = 12$	Substitution
$DE + 2\frac{3}{4} - 2\frac{3}{4} = 12 - 2\frac{3}{4}$	Subtract $2\frac{3}{4}$ from each side.
$DE = 9\frac{1}{4}$	Simplify.

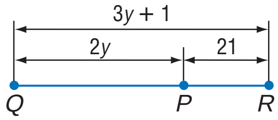
So, $\overline{DE}$ is $9\frac{1}{4}$ inches long.

Study Tip

Information from Figures
When no unit of measure is given on a figure, you can safely assume that all of the segments have the same unit of measure.

c. Find y and PQ if P is between Q and R, $PQ = 2y$, $QR = 3y + 1$, and $PR = 21$.

Draw a figure to represent this information.



$QR = QP + PR$

$3y + 1 = 2y + 21$ — Substitute known values.

$3y + 1 - 1 = 2y + 21 - 1$ — Subtract 1 from each side.

$3y = 2y + 20$ — Simplify.

$3y - 2y = 2y + 20 - 2y$ — Subtract $2y$ from each side.

$y = 20$ — Simplify.

$PQ = 2y$ — Given

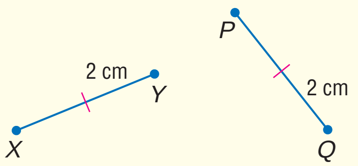
$PQ = 2(20)$ — $y = 20$

$PQ = 40$ — Multiply.

Look at the figure in part **a** of Example 4. Notice that $\overline{AB}$ and $\overline{BC}$ have the same measure. When segments have the same measure, they are said to be **congruent**.

Key Concept — *Congruent Segments*

- **Words** Two segments having the same measure are congruent.
- **Symbol** $\cong$ is read *is congruent to*. Red slashes on the figure also indicate that segments are congruent.
- **Model** $\overline{XY} \cong \overline{PQ}$



Study Tip

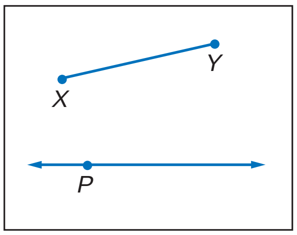
Constructions
A well-sharpened pencil will result in a more accurate construction.

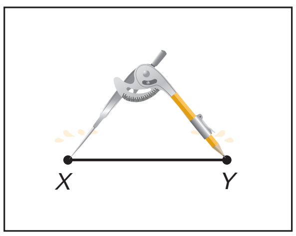
Constructions are methods of creating geometric figures without the benefit of measuring tools. Generally, only a pencil, straightedge, and compass are used in constructions. You can construct a segment that is congruent to a given segment by using a compass and straightedge.

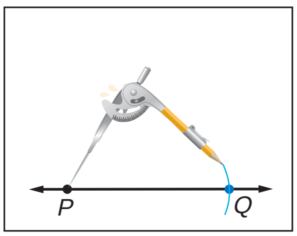
Construction

Copy a Segment

1. Draw a segment $\overline{XY}$. Elsewhere on your paper, draw a line and a point on the line. Label the point P.
2. Place the compass at point X and adjust the compass setting so that the pencil is at point Y.
3. Using that setting, place the compass point at P and draw an arc that intersects the line. Label the point of intersection Q. Because of identical compass settings, $\overline{PQ} \cong \overline{XY}$.

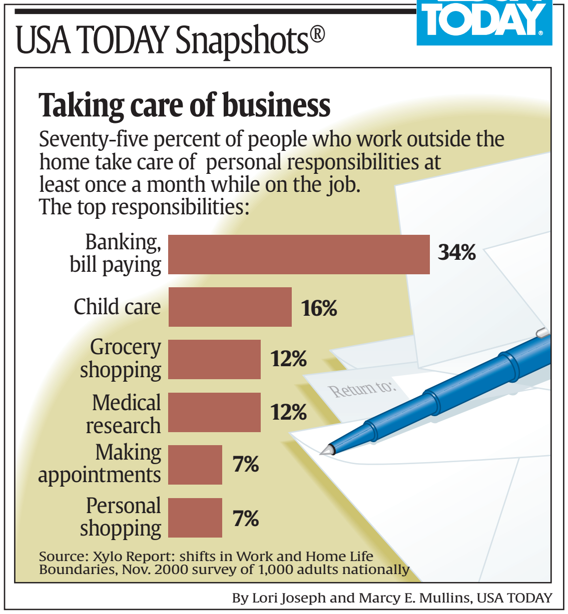






Example 5 Congruent Segments

TIME MANAGEMENT In the graph at the right, suppose a segment was drawn along the top of each bar. Which categories would have segments that are congruent? Explain.



The segments on the bars for grocery shopping and medical research would be congruent because they both have the same length, representing 12%.

The segments on bars for making appointments and personal shopping would be congruent because they have the same length, representing 7%.

Log on for:
- Updated data
- More activities on comparison using percent

www.geometryonline.com/usa_today

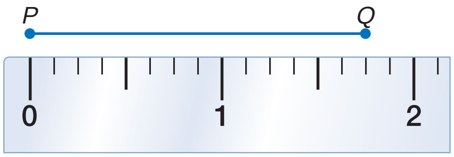
Check for Understanding

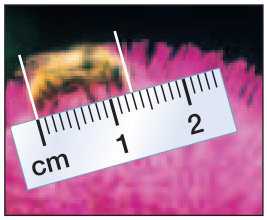
Concept Check

1. **Describe** how to measure a segment with a ruler that is divided into eighths of an inch.
2. **OPEN ENDED** Name or draw some geometric figures that have congruent segments.

Guided Practice

Find the length of each line segment or object.

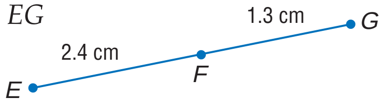
3. 

4. 

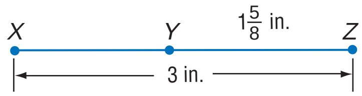
5. Find the precision for a measurement of 14 meters. Explain its meaning.
6. Find the precision for a measurement of $3\frac{1}{4}$ inches. Explain its meaning.

Find the measurement of each segment. Assume that each figure is not drawn to scale.

7. $\overline{EG}$



8. $\overline{XY}$

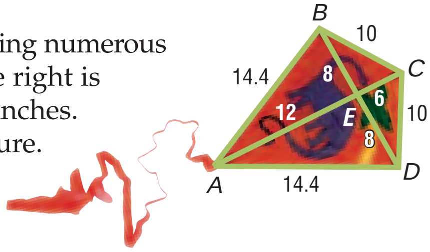


Find the value of the variable and *LM* if *L* is between *N* and *M*.

9. $NL = 5x$, $LM = 3x$, and $NL = 15$
10. $NL = 6x - 5$, $LM = 2x + 3$, and $NM = 30$

Application

11. **KITES** Kite making has become an art form using numerous shapes and designs for flight. The figure at the right is known as a *diamond kite*. The measures are in inches. Name all of the congruent segments in the figure.



(t)Rich Brommer, (b)C.W. McKeen/Syracuse Newspapers/The Image Works

Practice and Apply

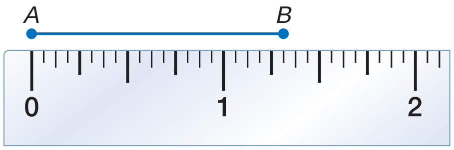
Homework Help

For Exercises	See Examples
12–15	1, 2
16–21	3
22–33	4
34–39	5

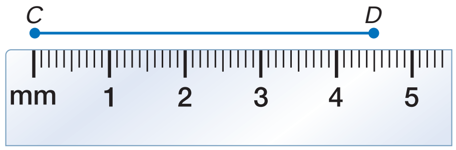
Extra Practice
See page 754.

Find the length of each line segment or object.

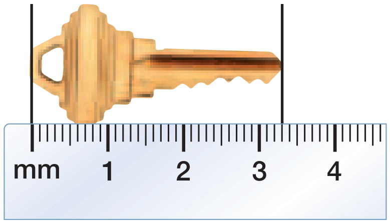
12.



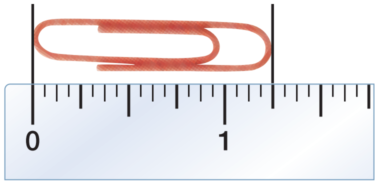
13.



14.



15.



Find the precision for each measurement. Explain its meaning.

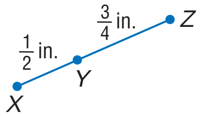
16. 80 in. **17.** 22 mm **18.** $16\frac{1}{2}$ in.

19. 308 cm **20.** 3.75 meters **21.** $3\frac{1}{4}$ ft

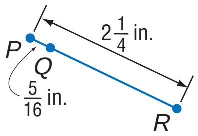
Find the measurement of each segment.

22. *AC*

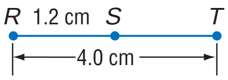
23. *XZ*



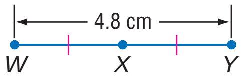
24. *QR*



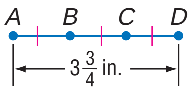
25. *ST*



26. *WX*



27. *BC*



Find the value of the variable and *ST* if *S* is between *R* and *T*.

28. $RS = 7a$, $ST = 12a$, $RS = 28$

29. $RS = 12$, $ST = 2x$, $RT = 34$

30. $RS = 2x$, $ST = 3x$, $RT = 25$

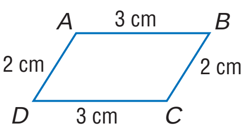
31. $RS = 16$, $ST = 2x$, $RT = 5x + 10$

32. $RS = 3y + 1$, $ST = 2y$, $RT = 21$

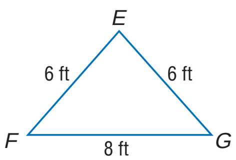
33. $RS = 4y - 1$, $ST = 2y - 1$, $RT = 5y$

Use the figures to determine whether each pair of segments is congruent.

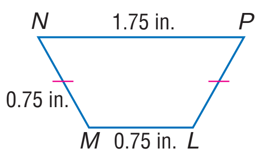
34. $\overline{AB}$, $\overline{CD}$



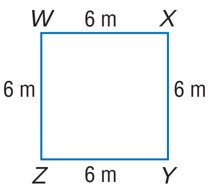
35. $\overline{EF}$, $\overline{FG}$



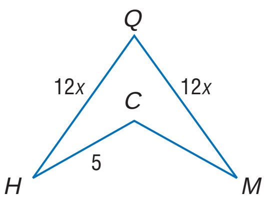
36. $\overline{NP}$, $\overline{LM}$



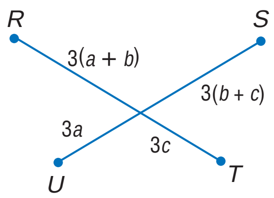
37. $\overline{WX}$, $\overline{XY}$



38. $\overline{CH}$, $\overline{CM}$

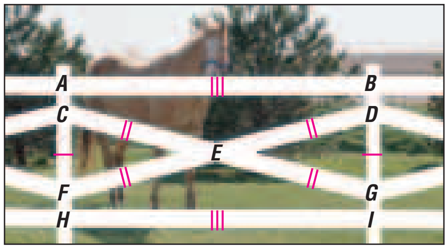


39. $\overline{TR}$, $\overline{SU}$



40. **MUSIC** A CD has a single spiral track of data, circling from the inside of the disc to the outside. Use a metric ruler to determine the full width of a music CD.

41. **DOORS** Name all segments in the crossbuck pattern in the picture that appear to be congruent.



42. **CRAFTS** Martin makes pewter figurines and wants to know how much molten pewter he needs for each mold. He knows that when a solid object with a volume of 1 cubic centimeter is submerged in water, the water level rises 1 milliliter. Martin pours 200 mL of water in a measuring cup, completely submerges a figurine in it, and watches it rise to 343 mL. What is the maximum amount of molten pewter, in cubic centimeters, Martin would need to make a figurine? Explain.

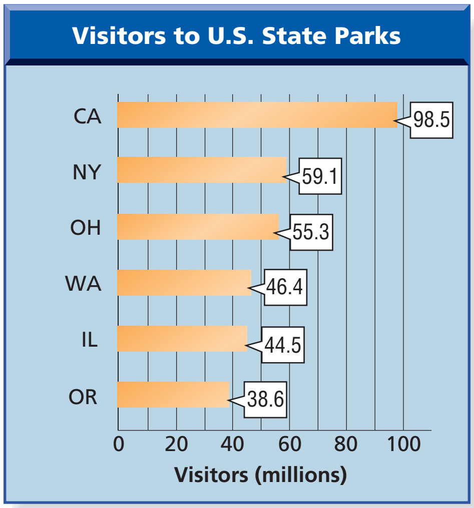
More About. . .

Recreation

There are more than 3300 state parks, historic sites, and natural areas in the United States. Most of the parks are open year round to visitors.

Source: *Parks Directory of the United States*

RECREATION **For Exercises 43–45, refer to the graph that shows the states with the greatest number of visitors to state parks in a recent year.**



Source: National Association of Park Directors

43. To what number can the precision of the data be measured?

44. Find the precision for the California data.

45. Can you be sure that 1.9 million more people visited Washington state parks than Illinois state parks? Explain.

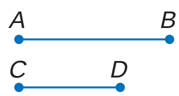
Online Research **Data Update** Find the current park data for your state and determine the precision of its measure. Visit www.geometryonline.com/data_update to learn more.

PERIMETER **For Exercises 46 and 47, use the following information.**
The **perimeter** of a geometric figure is the sum of the lengths of its sides. Pablo used a ruler divided into centimeters and measured the sides of a triangle as 3 centimeters, 5 centimeters, and 6 centimeters. Use what you know about the accuracy of any measurement to answer each question.

46. What is the least possible perimeter of the triangle? Explain.

47. What is the greatest possible perimeter of the triangle? Explain.

CONSTRUCTION **For Exercises 48 and 49, refer to the figure.**



48. Construct a segment whose measure is $4(CD)$.

49. Construct a segment that has length $3(AB) - 2(CD)$.

50. **CRITICAL THINKING** **Significant digits** represent the accuracy of a measurement.
- Nonzero digits are always significant.
- In whole numbers, zeros are significant if they fall between nonzero digits.
- In decimal numbers greater than or equal to 1, every digit is significant.
- In decimal numbers less than 1, the first nonzero digit and every digit to its right are significant.

For example, 600.070 has six significant digits, but 0.0210 has only three. How many significant digits are there in each measurement below?

a. 83,000 miles **b.** 33,002 miles **c.** 450.0200 liters

51. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why are units of measure important?

Include the following in your answer.

- an example of how measurements might be misinterpreted, and
- what measurements you can assume from a figure.

Extending the Lesson

RELATIVE ERROR Accuracy is an indication of **relative error**. The relative error is the ratio of the half-unit difference in precision to the entire measure. The relative error is expressed as a percent. The smaller the relative error of a measurement, the more accurate the measure is. For 11 inches, the relative error can be found as follows.

$$\frac{\text{allowable error}}{\text{measure}} = \frac{0.5 \text{ in.}}{11 \text{ in.}} \approx 0.045 \text{ or } 4.5\% \quad \text{Divide and convert to percent.}$$

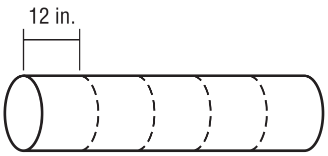
Determine the relative error for each measurement.

52. 27 ft
53. $14\frac{1}{2}$ in.
54. 42.3 cm
55. 63.7 km

Standardized Test Practice

56. The pipe shown is divided into five equal sections. How many feet long is the pipe?

(A) 2.4 ft (B) 5 ft
(C) 28.8 ft (D) 60 ft



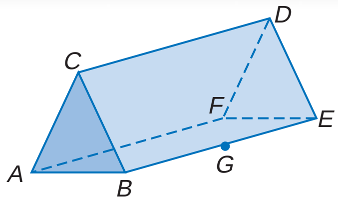
57. **ALGEBRA** Forty percent of a collection of 80 tapes are jazz tapes, and the rest are blues tapes. How many blues tapes are in the collection?

(A) 32 (B) 40 (C) 42 (D) 48

Maintain Your Skills

Mixed Review

Refer to the figure at the right. *(Lesson 1-1)*



58. Name three collinear points.
59. Name two planes that contain points *B* and *C*.
60. Name another point in plane *DFA*.
61. How many planes are shown?

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression if $a = 3$, $b = 8$, and $c = 2$.**

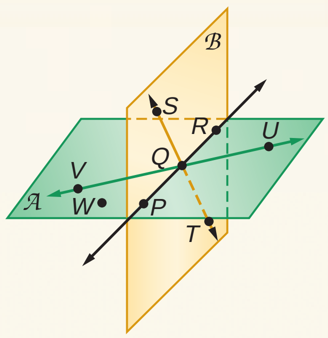
(To review ***evaluating expressions****, see page 736.)*

62. $2a + 2b$
63. $ac + bc$
64. $\frac{a - c}{2}$
65. $\sqrt{(c - a)^2}$

Practice Quiz 1 — *Lessons 1-1 and 1-2*

For Exercises 1–3, refer to the figure. *(Lesson 1-1)*

1. Name the intersection of planes $\mathcal{A}$ and $\mathcal{B}$.
2. Name another point that is collinear with points *S* and *Q*.
3. Name a line that is coplanar with $\overleftrightarrow{VU}$ and point *W*.



Given that *R* is between *S* and *T*, find each measure. *(Lesson 1-2)*

4. $RS = 6$, $TR = 4.5$, $TS =$ __?__.
5. $TS = 11.75$, $TR = 3.4$, $RS =$ __?__.

Probability and Segment Measure

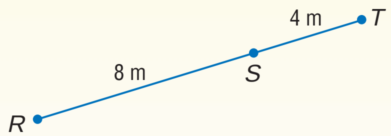
You may remember that probability is often expressed as a fraction.

$$\text{Probability } (P) \text{ of an event} = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

To find the probability that a point lies on a segment, you need to calculate the length of the segment.

Activity

Assume that point Q is contained in $\overline{RT}$. Find the probability that Q is contained in $\overline{RS}$.



Collect Data

- Find the measures of all segments in the figure.
- $RS = 8$ and $ST = 4$, so $RT = RS + ST$ or 12.
- While a point has no dimension, the segment that contains it does have one dimension, length. To calculate the probability that a point, randomly selected, is in a segment contained by another segment, you must compare their lengths.

$$P(Q \text{ lies in } \overline{RS}) = \frac{RS}{RT}$$

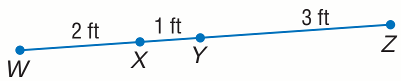
$$= \frac{8}{12} \quad RS = 8 \text{ and } RT = 12$$

$$= \frac{2}{3} \quad \text{Simplify.}$$

The probability that Q is contained in $\overline{RS}$ is $\frac{2}{3}$.

Analyze

For Exercises 1–3, refer to the figure at the right.



1. Point J is contained in $\overline{WZ}$. What is the probability that J is contained in $\overline{XY}$?

2. Point R is contained in $\overline{WZ}$. What is the probability that R is contained in $\overline{YZ}$?

3. Point S is contained in $\overline{WY}$. What is the probability that S is contained in $\overline{XY}$?

Make a Conjecture

For Exercises 4–5, refer to the figure for Exercises 1–3.

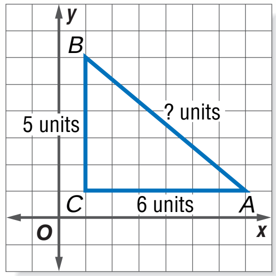
4. Point T is contained in both $\overline{WY}$ and $\overline{XZ}$. What do you think is the probability that T is contained in $\overline{XY}$? Explain.

5. Point U is contained in $\overline{WX}$. What do you think is the probability that U is contained in $\overline{YZ}$? Explain.

1-3 Distance and Midpoints

What You'll Learn

- Find the distance between two points.
- Find the midpoint of a segment.



Vocabulary

- midpoint
- segment bisector

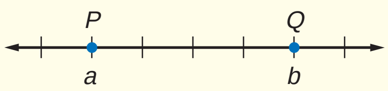
How can you find the distance between two points without a ruler?

Whenever you connect two points on a number line or on a plane, you have graphed a line segment. Distance on a number line is determined by counting the units between the two points. On a coordinate plane, you can use the Pythagorean Theorem to find the distance between two points. In the figure, to find the distance from A to B, use $(AC)^2 + (CB)^2 = (AB)^2$.

DISTANCE BETWEEN TWO POINTS The coordinates of the endpoints of a segment can be used to find the length of the segment. Because the distance from A to B is the same as the distance from B to A, the order in which you name the endpoints makes no difference.

Key Concept — Distance Formulas

- **Number line**



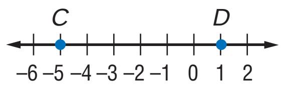
$PQ = |b - a|$ or $|a - b|$

- **Coordinate Plane**

The distance d between two points with coordinates (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Example 1 Find Distance on a Number Line

Use the number line to find CD.



The coordinates of C and D are −5 and 1.

$CD = |-5 - 1|$ — Distance Formula

$= |-6|$ or 6 — Simplify.

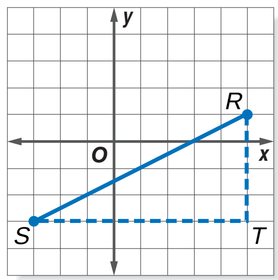
Study Tip

Pythagorean Theorem
Recall that the Pythagorean Theorem is often expressed as $a^2 + b^2 = c^2$, where a and b are the measures of the shorter sides (legs) of a right triangle, and c is the measure of the longest side (hypotenuse) of a right triangle.

You can use the Pythagorean Theorem to find the distance between two points on a coordinate plane.

Example 2 Find Distance on a Coordinate Plane

Find the distance between $R(5, 1)$ and $S(-3, -3)$.



Method 1 Pythagorean Theorem

Use the gridlines to form a triangle so you can use the Pythagorean Theorem.

$(RS)^2 = (RT)^2 + (ST)^2$ — Pythagorean Theorem

$(RS)^2 = 4^2 + 8^2$ — $RT = 4$ units, $ST = 8$ units

$(RS)^2 = 80$ — Simplify.

$RS = \sqrt{80}$ — Take the square root of each side.

Study Tip

Distance Formula
The Pythagorean Theorem is used to develop the Distance Formula. You will learn more about the Pythagorean Theorem in Lesson 7-2.

Method 2 Distance Formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$RS = \sqrt{(-3 - 5)^2 + (-3 - 1)^2}$	$(x_1, y_1) = (5, 1)$ and $(x_2, y_2) = (-3, -3)$
$RS = \sqrt{(-8)^2 + (-4)^2}$	Simplify.
$RS = \sqrt{80}$	Simplify.

The distance from R to S is $\sqrt{80}$ units. You can use a calculator to find that $\sqrt{80}$ is approximately 8.94.

MIDPOINT OF A SEGMENT

The **midpoint** of a segment is the point halfway between the endpoints of the segment. If X is the midpoint of $\overline{AB}$, then $AX = XB$.

Geometry Activity

Midpoint of a Segment

Model

- Graph points $A(5, 5)$ and $B(-1, 5)$ on grid paper. Draw $\overline{AB}$.
- Hold the paper up to the light and fold the paper so that points A and B match exactly. Crease the paper slightly.
- Open the paper and put a point where the crease intersects $\overline{AB}$. Label this midpoint as C.
- Repeat the first three steps using endpoints $X(-4, 3)$ and $Y(2, 7)$. Label the midpoint Z.

Make a Conjecture

1. What are the coordinates of point C?
2. What are the lengths of $\overline{AC}$ and $\overline{CB}$?
3. What are the coordinates of point Z?
4. What are the lengths of $\overline{XZ}$ and $\overline{ZY}$?
5. Study the coordinates of points A, B, and C. Write a rule that relates these coordinates. Then use points X, Y, and Z to verify your conjecture.

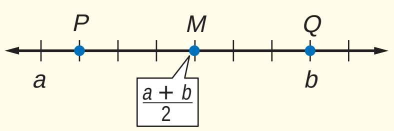
The points found in the activity are both midpoints of their respective segments.

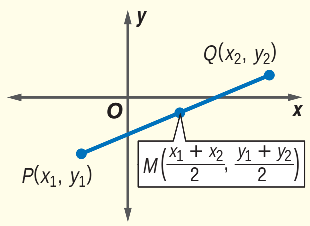
Study Tip

Common Misconception
The Distance Formula and the Midpoint Formula do not use the same relationship among the coordinates.

Key Concept — Midpoint

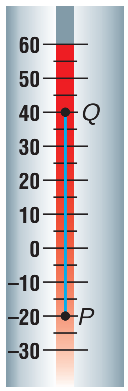
	Number Line	Coordinate Plane
Words	The midpoint M of $\overline{PQ}$ is the point between P and Q such that $PM = MQ$.	
Symbols	The coordinate of the midpoint of a segment whose endpoints have coordinates a and b is $\frac{a+b}{2}$.	The coordinates of the midpoint of a segment whose endpoints have coordinates (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.
Models		





Example 3 Find Coordinates of Midpoint

a. TEMPERATURE Find the coordinate of the midpoint of $\overline{PQ}$.



The coordinates of P and Q are -20 and 40.

Let M be the midpoint of $\overline{PQ}$.

$$M = \frac{-20 + 40}{2} \quad a = -20, b = 40$$

$$= \frac{20}{2} \text{ or } 10 \quad \text{Simplify.}$$

b. Find the coordinates of M, the midpoint of $\overline{PQ}$, for $P(-1, 2)$ and $Q(6, 1)$.

Let P be (x_1, y_1) and Q be (x_2, y_2).

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = M\left(\frac{-1 + 6}{2}, \frac{2 + 1}{2}\right) \quad (x_1, y_1) = (-1, 2), (x_2, y_2) = (6, 1)$$

$$= M\left(\frac{5}{2}, \frac{3}{2}\right) \text{ or } M\left(2\tfrac{1}{2}, 1\tfrac{1}{2}\right) \quad \text{Simplify.}$$

WebQuest

Latitude and longitude form another coordinate system. The latitude, longitude, degree distance, and monthly high temperature can be used to create several different scatter plots. Visit www.geometry online.com/webquest to continue work on your WebQuest project.

You can also find the coordinates of the endpoint of a segment if you know the coordinates of its other endpoint and its midpoint.

Example 4 Find Coordinates of Endpoint

Find the coordinates of X if $Y(-2, 2)$ is the midpoint of $\overline{XZ}$ and Z has coordinates $(2, 8)$.

Let Z be (x_2, y_2) in the Midpoint Formula.

$$Y(-2, 2) = Y\left(\frac{x_1 + 2}{2}, \frac{y_1 + 8}{2}\right) \quad (x_2, y_2) = (2, 8)$$

Write two equations to find the coordinates of X.

$$-2 = \frac{x_1 + 2}{2}$$

$-4 = x_1 + 2$ Multiply each side by 2.

$-6 = x_1$ Subtract 2 from each side.

$$2 = \frac{y_1 + 8}{2}$$

$4 = y_1 + 8$ Multiply each side by 2.

$-4 = y_1$ Subtract 8 from each side.

The coordinates of X are $(-6, -4)$.

Standardized Test Practice

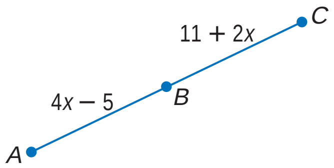
Example 5 Use Algebra to Find Measures

Multiple-Choice Test Item

What is the measure of $\overline{BC}$ if B is the midpoint of $\overline{AC}$?

(A) -5 (B) 8

(C) 17 (D) 27



Read the Test Item

You know that B is the midpoint of $\overline{AC}$, and the figure gives algebraic measures for $\overline{AB}$ and $\overline{BC}$. You are asked to find the measure of $\overline{BC}$.

The Princeton Review

Test-Taking Tip

Eliminate Possibilities You can sometimes eliminate choices by looking at the reasonableness of the answer. In this test item, you can eliminate choice A because measures cannot be negative.

(continued on the next page)

Solve the Test Item

Because B is the midpoint, you know that $AB = BC$. Use this equation and the algebraic measures to find a value for x.

$AB = BC$	Definition of midpoint
$4x - 5 = 11 + 2x$	$AB = 4x - 5$, $BC = 11 + 2x$
$4x = 16 + 2x$	Add 5 to each side.
$2x = 16$	Subtract $2x$ from each side.
$x = 8$	Divide each side by 2.

Now substitute 8 for x in the expression for BC.

$BC = 11 + 2x$	Original measure
$BC = 11 + 2(8)$	$x = 8$
$BC = 11 + 16$ or 27	Simplify.

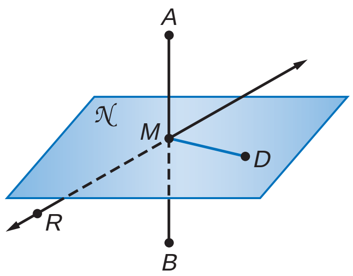
The answer is D.

Study Tip

Segment Bisectors

There can be an infinite number of bisectors, and each must contain the midpoint of the segment.

Any segment, line, or plane that intersects a segment at its midpoint is called a **segment bisector**. In the figure at the right, M is the midpoint of $\overline{AB}$. Plane $\mathcal{N}$, $\overline{MD}$, $\overleftrightarrow{RM}$, and point M are all bisectors of $\overline{AB}$. We say that they *bisect* $\overline{AB}$.

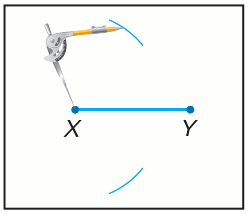


You can construct a line that bisects a segment without measuring to find the midpoint of the given segment.

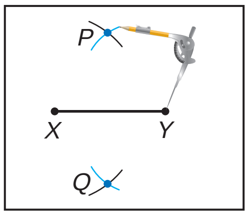
Construction

Bisect a Segment

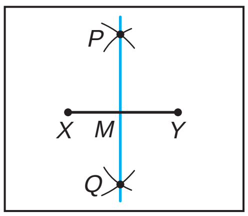
1. Draw a segment and name it $\overline{XY}$. Place the compass at point X. Adjust the compass so that its width is greater than $\frac{1}{2}XY$. Draw arcs above and below $\overline{XY}$.



2. Using the same compass setting, place the compass at point Y and draw arcs above and below $\overline{XY}$ intersect the two arcs previously drawn. Label the points of the intersection of the arcs as P and Q.



3. Use a straightedge to draw $\overline{PQ}$. Label the point where it intersects $\overline{XY}$ as M. Point M is the midpoint of $\overline{XY}$, and $\overline{PQ}$ is a bisector of $\overline{XY}$. Also $XM = MY = \frac{1}{2}XY$.



Check for Understanding

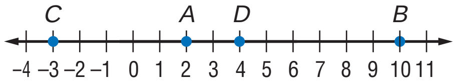
Concept Check

1. **Explain** three ways to find the midpoint of a segment.
2. **OPEN ENDED** Draw a segment. Construct the bisector of the segment and use a millimeter ruler to check the accuracy of your construction.

Guided Practice

Use the number line to find each measure.

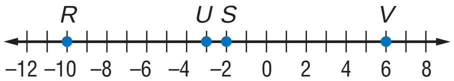
3. *AB*
4. *CD*



5. Use the Pythagorean Theorem to find the distance between $X(7, 11)$ and $Y(-1, 5)$.
6. Use the Distance Formula to find the distance between $D(2, 0)$ and $E(8, 6)$.

Use the number line to find the coordinate of the midpoint of each segment.

7. $\overline{RS}$
8. $\overline{UV}$



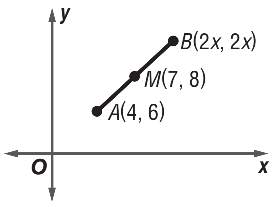
Find the coordinates of the midpoint of a segment having the given endpoints.

9. $X(-4, 3), Y(-1, 5)$
10. $A(2, 8), B(-2, 2)$
11. Find the coordinates of A if $B(0, 5.5)$ is the midpoint of $\overline{AC}$ and C has coordinates $(-3, 6)$.

Standardized Test Practice

12. Point M is the midpoint of $\overline{AB}$. What is the value of x in the figure?

(A) 1.5 (B) 5
(C) 5.5 (D) 11



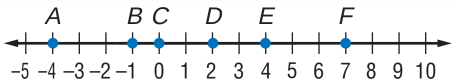
Practice and Apply

Homework Help

For Exercises	See Examples
13–18	1
19–28	2
29, 30	5
31–42	3
43–45	4

Extra Practice
See page 754.

Use the number line to find each measure.



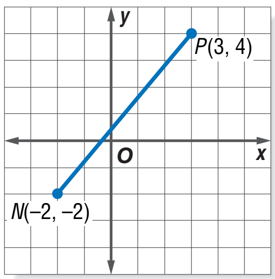
13. *DE*
14. *CF*
15. *AB*
16. *AC*
17. *AF*
18. *BE*

Use the Pythagorean Theorem to find the distance between each pair of points.

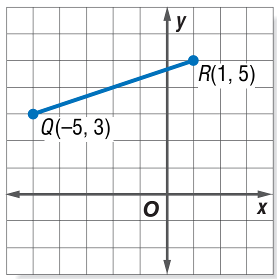
19. $A(0, 0), B(8, 6)$
20. $C(-10, 2), D(-7, 6)$
21. $E(-2, -1), F(3, 11)$
22. $G(-2, -6), H(6, 9)$

Use the Distance Formula to find the distance between each pair of points.

23. $J(0, 0), K(12, 9)$
24. $L(3, 5), M(7, 9)$
25. $S(-3, 2), T(6, 5)$
26. $U(2, 3), V(5, 7)$
27.



28.

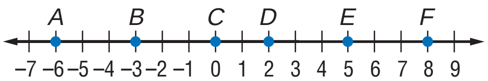


PERIMETER For Exercises 29 and 30, use the following information.
The perimeter of a figure is the sum of the lengths of its sides.

29. The vertices of a triangle are located at $X(-2, -1)$, $Y(2, 5)$, and $Z(4, 3)$. What is the perimeter of this triangle? Round to the nearest tenth.

30. What is the perimeter of a square whose vertices are $A(-4, -3)$, $B(-5, 1)$, $C(-1, 2)$, and $D(0, -2)$?

Use the number line to find the coordinate of the midpoint of each segment.



31. $\overline{AC}$ **32.** $\overline{DF}$ **33.** $\overline{CE}$

34. $\overline{BD}$ **35.** $\overline{AF}$ **36.** $\overline{BE}$

Find the coordinates of the midpoint of a segment having the given endpoints.

37. $A(8, 4)$, $B(12, 2)$ **38.** $C(9, 5)$, $D(17, 4)$

39. $E(-11, -4)$, $F(-9, -2)$ **40.** $G(4, 2)$, $H(8, -6)$

41. $J(3.4, 2.1)$, $K(7.8, 3.6)$ **42.** $L(-1.4, 3.2)$, $M(2.6, -5.4)$

Find the coordinates of the missing endpoint given that S is the midpoint of $\overline{RT}$.

43. $T(-4, 3)$, $S(-1, 5)$ **44.** $T(2, 8)$, $S(-2, 2)$ **45.** $R\left(\frac{2}{3}, -5\right)$, $S\left(\frac{5}{3}, 3\right)$

Study Tip

Latitude and Longitude
Actual distances on Earth are calculated along the curve of Earth's surface. However, when the two points are located relatively close together, you can apply the concepts of plane geometry to approximate coordinates and distances.

GEOGRAPHY For Exercises 46 and 47, use the following information.
The geographic center of Texas is located northeast of Brady at (31.1°, 99.3°), which represent north latitude and west longitude. El Paso is located near the western border of Texas at (31.8°, 106.4°).

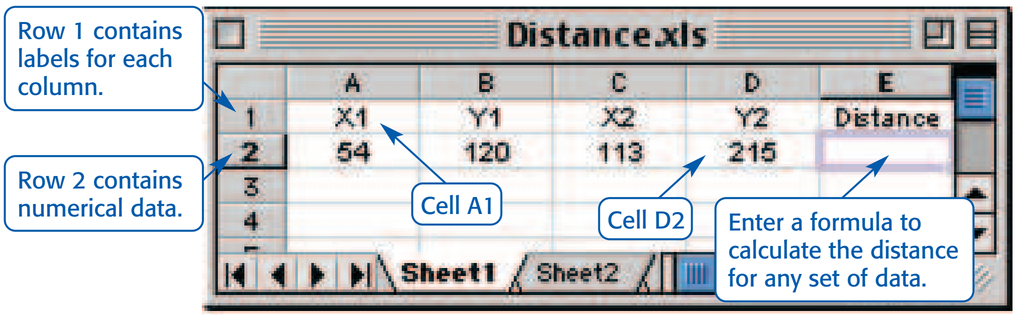
46. If El Paso is one endpoint of a segment and the geographic center is its midpoint, find the latitude and longitude of the other endpoint.

47. Use an atlas or the Internet to find a city near this location.

Study Tip

Spreadsheets
Spreadsheets often use special commands to perform operations. For example, to find the square root of a number, use the SQRT command. To raise a number to a power, x^2 for example, write it as x^2.

SPREADSHEETS For Exercises 48 and 49, refer to the information at the left and use the following information.
Spreadsheets can be used to perform calculations quickly. Values are used in formulas by using a specific cell name. For example, the value of x_1 below is used in a formula using its cell name, A2. Special commands are used to perform some operations. For example, $\sqrt{x_1 - x_2}$ would be written as =SQRT(A2 − C2). The spreadsheet below can be used to calculate the distance between two points.



48. Write a formula for cell E2 that could be used to calculate the distance between (x_1, y_1) and (x_2, y_2).

49. Find the distance between each pair of points to the nearest tenth.

a. (54, 120), (113, 215) **b.** (68, 153), (175, 336)

c. (421, 454), (502, 798) **d.** (837, 980), (612, 625)

e. (1967, 3), (1998, 24) **f.** (4173.5, 34.9), (2080.6, 22.4)

ENLARGEMENT **For Exercises 50–53, use the following information.**
The coordinates of the vertices of a triangle are $A(1, 3)$, $B(6, 10)$, and $C(11, 18)$.

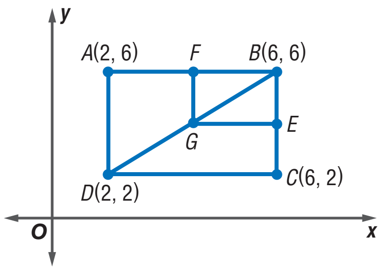
50. Find the perimeter of $\triangle ABC$.

51. Suppose each coordinate is multiplied by 2. What is the perimeter of this triangle?

52. Find the perimeter of the triangle when the coordinates are multiplied by 3.

53. Make a conjecture about the perimeter of a triangle when the coordinates of its vertices are multiplied by the same positive factor.

54. **CRITICAL THINKING** In the figure, $\overline{GE}$ bisects $\overline{BC}$, and $\overline{GF}$ bisects $\overline{AB}$. $\overline{GE}$ is a horizontal segment, and $\overline{GF}$ is a vertical segment.



a. Find the coordinates of points F and E.

b. Name the coordinates of G and explain how you calculated them.

c. Describe what relationship, if any, exists between $\overline{DG}$ and $\overline{GB}$. Explain.

55. **CRITICAL THINKING** $\overline{WZ}$ has endpoints $W(-3, -8)$ and $Z(5, 12)$. Point X lies between W and Z, such that $WX = \frac{1}{4}WZ$. Find the coordinates of X.

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you find the distance between two points without a ruler?

Include the following in your answer:

- how to use the Pythagorean Theorem and the Distance Formula to find the distance between two points, and
- the length of $\overline{AB}$ from the figure on page 21.

Standardized Test Practice

57. Find the distance between points at $(6, 11)$ and $(-2, -4)$.

(A) 16 units (B) 17 units (C) 18 units (D) 19 units

58. **ALGEBRA** Which equation represents the following problem?

Fifteen minus three times a number equals negative twenty-two. Find the number.

(A) $15 - 3n = -22$ (B) $3n - 15 = -22$

(C) $3(15 - n) = -22$ (D) $3(n - 15) = -22$

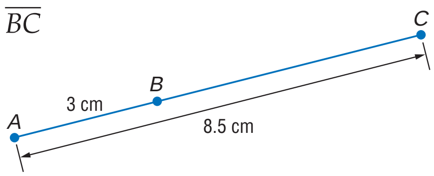
Maintain Your Skills

Mixed Review

Find the measurement of each segment. *(Lesson 1-2)*

59. $\overline{WY}$

60. $\overline{BC}$



Draw and label a figure for each relationship. *(Lesson 1-1)*

61. four noncollinear points A, B, C, and D that are coplanar

62. line m that intersects plane $\mathcal{A}$ and line n in plane $\mathcal{A}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation.** *(To review **solving equations**, see page 737.)*

63. $2k = 5k - 30$ **64.** $14x - 31 = 12x + 8$ **65.** $180 - 8t = 90 + 2t$

66. $12m + 7 = 3m + 52$ **67.** $8x + 7 = 5x + 20$ **68.** $13n - 18 = 5n + 32$

Geometry Activity

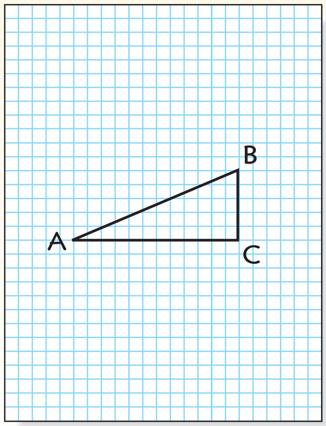
A Follow-Up of Lesson 1-3

Modeling the Pythagorean Theorem

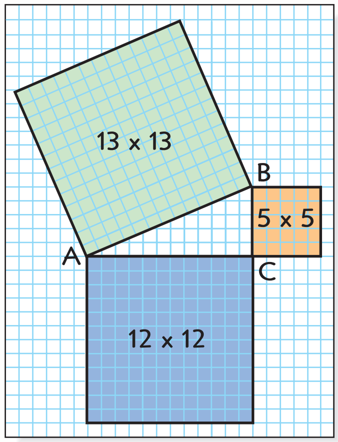
In Chapter 7, you will formally write a verification of the Pythagorean Theorem, but this activity will suggest that the Pythagorean Theorem holds for any right triangle. Remember that a right triangle is a triangle with a right angle, and that a right angle measures 90°.

Make a Model

- Draw right triangle *ABC* in the center of a piece of grid paper.



- Use another piece of grid paper to draw a square that is 5 units on each side, a square that is 12 units on each side, and a square that is 13 units on each side. Use colored pencils to shade each of these squares. Cut out the squares. Label them as 5 × 5, 12 × 12, and 13 × 13 respectively.
- Place the squares so that a side of the square matches up with a side of the right triangle.



Analyze

1. Determine the number of grid squares in each square you drew.

2. How do the numbers of grid squares relate?

3. If $AB = c$, $BC = a$, and $AC = b$, write an expression to describe each of the squares.

4. How does this expression compare with what you know about the Pythagorean Theorem?

Make a Conjecture

5. Repeat the activity for triangles with each of the side measures listed below. What do you find is true of the relationship of the squares on the sides of the triangle?

a. 3, 4, 5 **b.** 8, 15, 17 **c.** 6, 8, 10

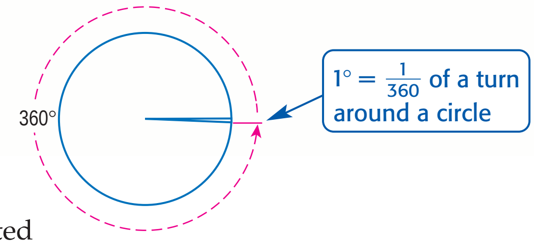
6. Repeat the activity with a right triangle whose shorter sides are both 5 units long. How could you determine the number of grid squares in the larger square?

1-4 Angle Measure

What You'll Learn

- Measure and classify angles.
- Identify and use congruent angles and the bisector of an angle.

How big is a degree?

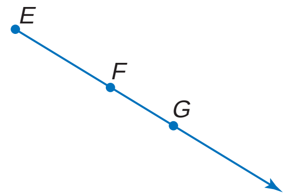


One of the first references to the measure now known as a degree came from astronomer Claudius Ptolemy. He based his observations of the solar system on a unit that resulted from dividing the circumference, or the distance around, a circle into 360 parts. This later became known as a **degree**. In this lesson, you will learn to measure angles in degrees.

Vocabulary

- degree
- ray
- opposite rays
- angle
- sides
- vertex
- interior
- exterior
- right angle
- acute angle
- obtuse angle
- angle bisector

MEASURE ANGLES A **ray** is part of a line. It has one endpoint and extends indefinitely in one direction. Rays are named stating the endpoint first and then any other point on the ray. The figure at the right shows ray *EF*, which can be symbolized as $\overrightarrow{EF}$. This ray could also be named as $\overrightarrow{EG}$, but not as $\overrightarrow{FE}$ because *F* is not the endpoint of the ray.



If you choose a point on a line, that point determines exactly two rays called **opposite rays**. Line *m*, shown below, is separated into two opposite rays, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. Point *P* is the common endpoint of those rays. $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are collinear rays.

Study Tip

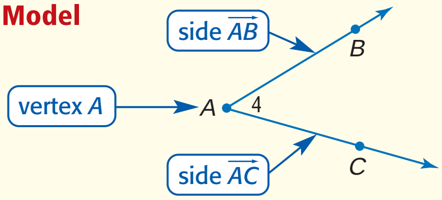
Reading Math

Opposite rays are also known as a *straight angle*. Its measure is 180°. Unless otherwise specified, the term *angle* in this book means a nonstraight angle.

An **angle** is formed by two *noncollinear* rays that have a common endpoint. The rays are called **sides** of the angle. The common endpoint is the **vertex**.

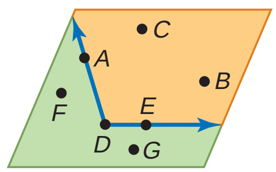
Key Concept — Angle

- **Words** An angle is formed by two noncollinear rays that have a common endpoint.
- **Symbols** $\angle A$
 $\angle BAC$
 $\angle CAB$
 $\angle 4$
- **Model**



An angle divides a plane into three distinct parts.

- Points *A*, *D*, and *E* lie on the angle.
- Points *C* and *B* lie in the **interior** of the angle.
- Points *F* and *G* lie in the **exterior** of the angle.

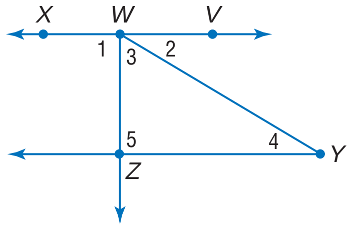


Study Tip

Naming Angles

You can name an angle by a single letter *only* when there is one angle shown at that vertex.

Example 1 Angles and Their Parts



a. Name all angles that have *W* as a vertex.

$\angle 1, \angle 2, \angle 3, \angle XWY, \angle ZWV$

b. Name the sides of $\angle 1$.

$\overrightarrow{WZ}$ and $\overrightarrow{WX}$ are the sides of $\angle 1$.

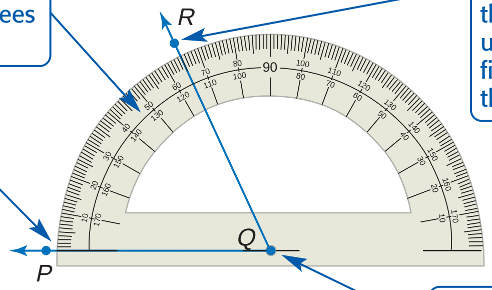
c. Write another name for $\angle WYZ$.

$\angle 4$, $\angle Y$, and $\angle ZYW$ are other names for $\angle WYZ$.

To measure an angle, you can use a *protractor*. Angle *PQR* is a 65 degree (65°) angle. We say that the *degree measure* of $\angle PQR$ is 65, or simply $m\angle PQR = 65$.

The protractor has two scales running from 0 to 180 degrees in opposite directions.

Align the 0 on either side of the scale with one side of the angle.



Since $\overrightarrow{QP}$ is aligned with the 0 on the outer scale, use the outer scale to find that $\overrightarrow{QR}$ intersects the scale at 65 degrees.

Place the center point of the protractor on the vertex.

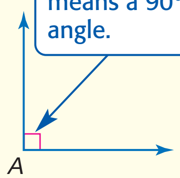
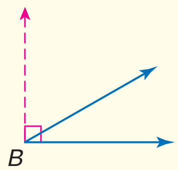
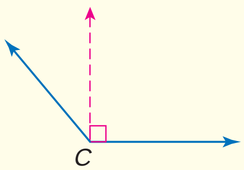
Angles can be classified by their measures.

Study Tip

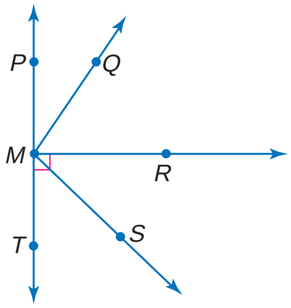
Classifying Angles

The corner of a piece of paper is a right angle. Use the corner to determine if an angle's measure is greater than 90 or less than 90.

Key Concept — Classify Angles

Name	right angle	acute angle	obtuse angle
Measure	$m\angle A = 90$	$m\angle B < 90$	$180 > m\angle C > 90$
Model	This symbol means a 90° angle. 		

Example 2 Measure and Classify Angles



Measure each angle named and classify it as *right*, *acute*, or *obtuse*.

a. $\angle PMQ$

Use a protractor to find that $m\angle PMQ = 30$.
$30 < 90$, so $\angle PMQ$ is an acute angle.

b. $\angle PMR$

$\angle PMR$ is marked with a right angle symbol, so measuring is not necessary; $m\angle PMR = 90$.

c. $\angle QMS$

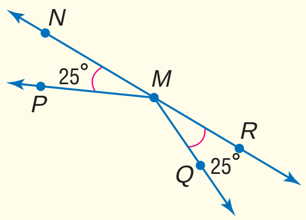
Use a protractor to find that $m\angle QMS = 110$. $\angle QMS$ is an obtuse angle.

CONGRUENT ANGLES

Just as segments that have the same measure are congruent, angles that have the same measure are congruent.

Key Concept — Congruent Angles

- **Words** Angles that have the same measure are congruent angles. Arcs on the figure also indicate which angles are congruent.
- **Symbols** $\angle NMP \cong \angle QMR$
- **Model**

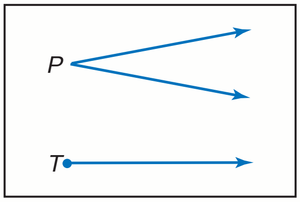


You can construct an angle congruent to a given angle without knowing the measure of the angle.

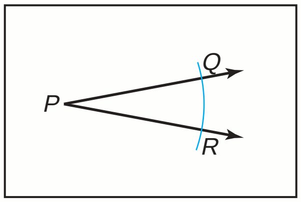
Construction

Copy an Angle

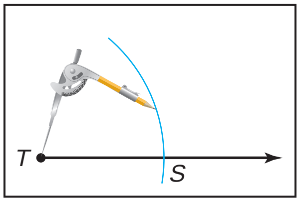
1. Draw an angle like $\angle P$ on your paper. Use a straightedge to draw a ray on your paper. Label its endpoint T.



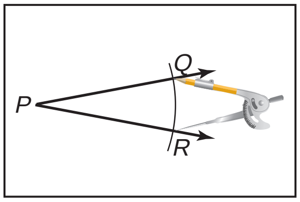
2. Place the tip of the compass at point P and draw a large arc that intersects both sides of $\angle P$. Label the points of intersection Q and R.



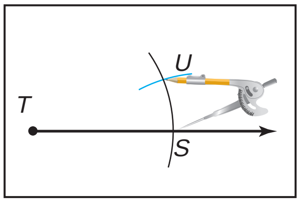
3. Using the same compass setting, put the compass at T and draw a large arc that intersects the ray. Label the point of intersection S.



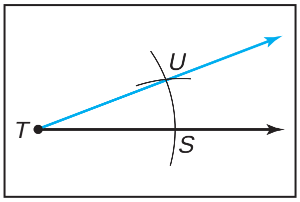
4. Place the point of your compass on R and adjust so that the pencil tip is on Q.



5. Without changing the setting, place the compass at S and draw an arc to intersect the larger arc you drew in Step 3. Label the point of intersection U.

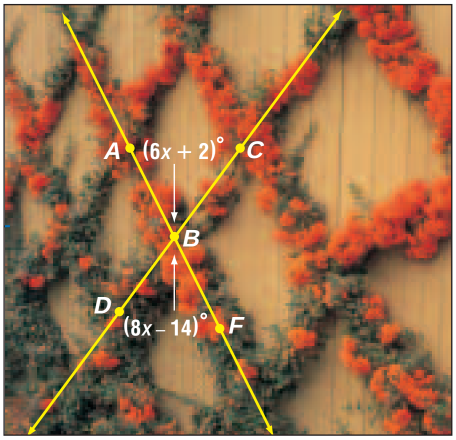


6. Use a straightedge to draw $\overrightarrow{TU}$.



Example 3 Use Algebra to Find Angle Measures

GARDENING **A trellis is often used to provide a frame for vining plants. Some of the angles formed by the slats of the trellis are congruent angles. In the figure, $\angle ABC \cong \angle DBF$. If $m\angle ABC = 6x + 2$ and $m\angle DBF = 8x - 14$, find the actual measurements of $\angle ABC$ and $\angle DBF$.**



$\angle ABC \cong \angle DBF$	Given
$m\angle ABC = m\angle DBF$	Definition of congruent angles
$6x + 2 = 8x - 14$	Substitution
$6x + 16 = 8x$	Add 14 to each side.
$16 = 2x$	Subtract 6x from each side.
$8 = x$	Divide each side by 2.

Use the value of x to find the measure of one angle.

$m\angle ABC = 6x + 2$	Given
$= 6(8) + 2$	$x = 8$
$= 48 + 2$ or 50	Simplify.

Since $m\angle ABC = m\angle DBF$, $m\angle DBF = 50$.

Both $\angle ABC$ and $\angle DBF$ measure 50°.

Study Tip

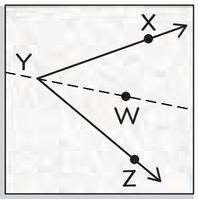
Checking Solutions

Check that you have computed the value of x correctly by substituting the value into the expression for $\angle DBF$. If you don't get the same measure as $\angle ABC$, you have made an error.

Geometry Activity

Bisect an Angle

Make a Model



- Draw any $\angle XYZ$ on patty paper or tracing paper.
- Fold the paper through point Y so that $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are aligned together.
- Open the paper and label a point on the crease in the interior of $\angle XYZ$ as point W.

Analyze the Model

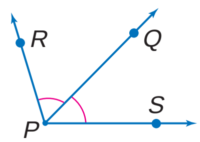
1. What seems to be true about $\angle XYW$ and $\angle WYZ$?
2. Measure $\angle XYZ$, $\angle XYW$, and $\angle WYZ$.
3. You learned about a segment bisector in Lesson 1-3. Write a sentence to explain the term *angle bisector*.

Study Tip

Adding Angle Measures

Just as with segments, when a line divides an angle into smaller angles, the sum of the measures of the smaller angles equals the measure of the largest angle. So in the figure, $m\angle RPS = m\angle RPQ + m\angle QPS$.

A ray that divides an angle into two congruent angles is called an **angle bisector**. If $\overrightarrow{PQ}$ is the angle bisector of $\angle RPS$, then point Q lies in the interior of $\angle RPS$ and $\angle RPQ \cong \angle QPS$.



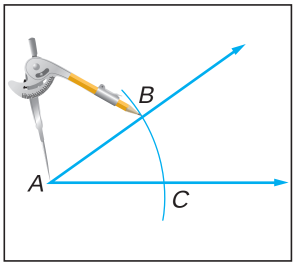
You can construct the angle bisector of any angle without knowing the measure of the angle.

Red Habegger/Grant Heilman Photography

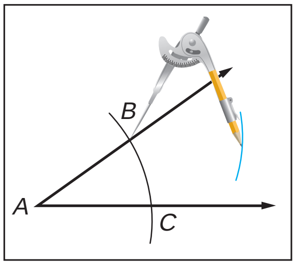
Construction

Bisect an Angle

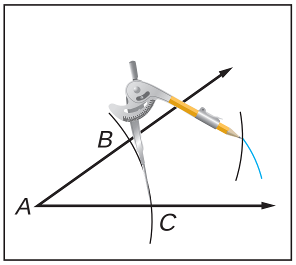
1. Draw an angle on your paper. Label the vertex as *A*. Put your compass at point *A* and draw a large arc that intersects both sides of ∠*A*. Label the points of intersection *B* and *C*.



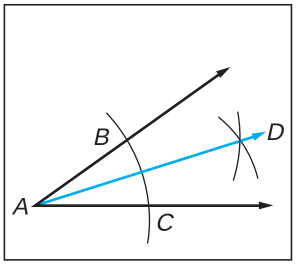
2. With the compass at point *B*, draw an arc in the interior of the angle.



3. Keeping the same compass setting, place the compass at point *C* and draw an arc that intersects the arc drawn in Step 2.



4. Label the point of intersection *D*. Draw $\overrightarrow{AD}$. $\overrightarrow{AD}$ is the bisector of ∠*A*. Thus, $m\angle BAD = m\angle DAC$ and $\angle BAD \cong \angle DAC$.



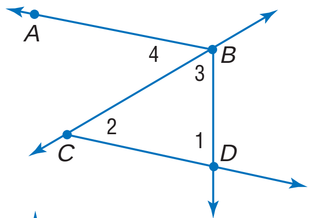
Check for Understanding

Concept Check

1. **Determine** whether all right angles are congruent.
2. **OPEN ENDED** Draw and label a figure to show $\overrightarrow{PR}$ that bisects ∠*SPQ* and $\overrightarrow{PT}$ that bisects ∠*SPR*. Use a protractor to measure each angle.
3. **Write** a statement about the measures of congruent angles *A* and *Z*.

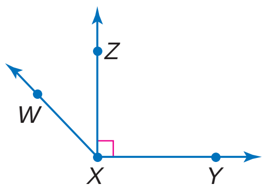
Guided Practice

For Exercises 4 and 5, use the figure at the right.



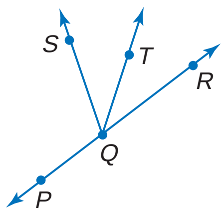
4. Name the vertex of ∠2.
5. Name the sides of ∠4.
6. Write another name for ∠*BDC*.

Measure each angle and classify as *right*, *acute*, or *obtuse*.



7. ∠*WXY*
8. ∠*WXZ*

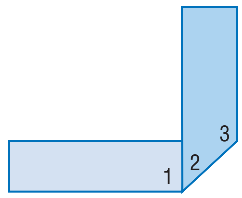
ALGEBRA **In the figure, $\overrightarrow{QP}$ and $\overrightarrow{QR}$ are opposite rays, and $\overrightarrow{QT}$ bisects ∠*RQS*.**



9. If $m\angle RQT = 6x + 5$ and $m\angle SQT = 7x - 2$, find $m\angle RQT$.
10. Find $m\angle TQS$ if $m\angle RQS = 22a - 11$ and $m\angle RQT = 12a - 8$.

Application

11. **ORIGAMI** The art of origami involves folding paper at different angles to create designs and three-dimensional figures. One of the folds in origami involves folding a strip of paper so that the lower edge of the strip forms a right angle with itself. Identify each numbered angle as *right*, *acute*, or *obtuse*.



Practice and Apply

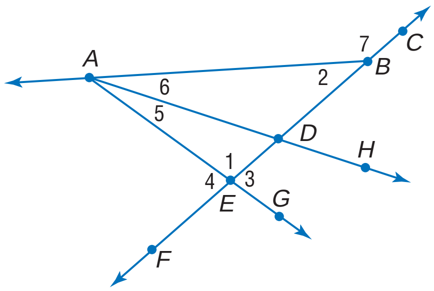
Homework Help

For Exercises	See Examples
12–27	1
28–33	2
34–39	3

Extra Practice
See page 755.

Name the vertex of each angle.

12. $\angle 1$ **13.** $\angle 2$
14. $\angle 6$ **15.** $\angle 5$



Name the sides of each angle.

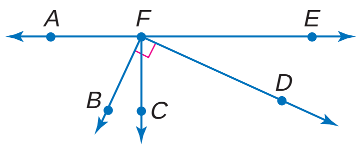
16. $\angle ADB$ **17.** $\angle 6$
18. $\angle 3$ **19.** $\angle 5$

Write another name for each angle.

20. $\angle 7$ **21.** $\angle AEF$
22. $\angle ABD$ **23.** $\angle 1$

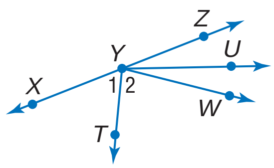
24. Name a point in the interior of $\angle GAB$.
25. Name an angle with vertex B that appears to be acute.
26. Name a pair of angles that share exactly one point.
27. If $\overrightarrow{AD}$ bisects $\angle EAB$ and $m\angle EAB = 60$, find $m\angle 5$ and $m\angle 6$.

Measure each angle and classify it as *right*, *acute*, or *obtuse*.



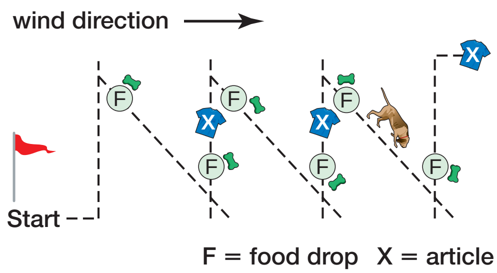
28. $\angle BFD$ **29.** $\angle AFB$
30. $\angle DFE$ **31.** $\angle EFC$
32. $\angle AFD$ **33.** $\angle EFB$

ALGEBRA **In the figure, $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are opposite rays. $\overrightarrow{YU}$ bisects $\angle ZYW$, and $\overrightarrow{YT}$ bisects $\angle XYW$.**



34. If $m\angle ZYU = 8p - 10$ and $m\angle UYW = 10p - 20$, find $m\angle ZYU$.
35. If $m\angle 1 = 5x + 10$ and $m\angle 2 = 8x - 23$, find $m\angle 2$.
36. If $m\angle 1 = y$ and $m\angle XYW = 6y - 24$, find y.
37. If $m\angle WYZ = 82$ and $m\angle ZYU = 4r + 25$, find r.
38. If $m\angle WYX = 2(12b + 7)$ and $m\angle ZYU = 9b - 1$, find $m\angle UYW$.
39. If $\angle ZYW$ is a right angle and $m\angle ZYU = 13a - 7$, find a.

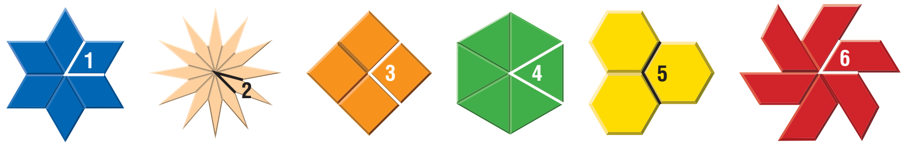
40. **DOG TRACKING** A dog is *tracking* when it is following the scent trail left by a human being or other animal that has passed along a certain route. One of the training exercises for these dogs is a tracking trail. The one shown is called an acute tracking trail. Explain why it might be called this.



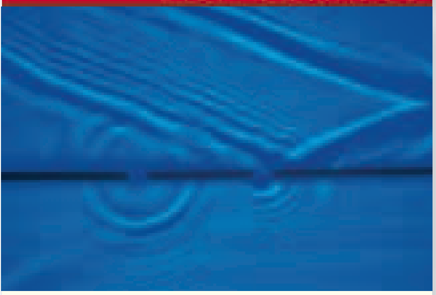
41. **LANGUAGE** The words *obtuse* and *acute* have other meanings in the English language. Look these words up and write how the everyday meaning relates to the mathematical meaning.

CONTENTS

42. PATTERN BLOCKS Pattern blocks can be arranged to fit in a circular pattern without leaving spaces. Remember that the measurement around a full circle is 360°. Determine the angle measure of the numbered angles shown below.



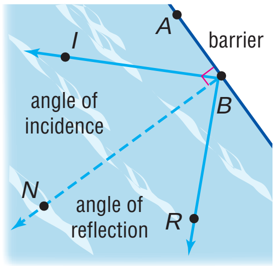
More About. . .



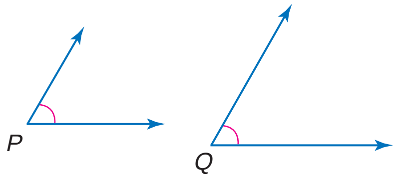
Physics

A ripple tank is a large glass-bottomed tank of water. A light is placed above the water, and a white sheet of paper is placed below the tank. Because rays of light undergo bending as they pass through the troughs and crests of the water, there is a pattern of light and dark spots on the white sheet of paper. These model the wave.

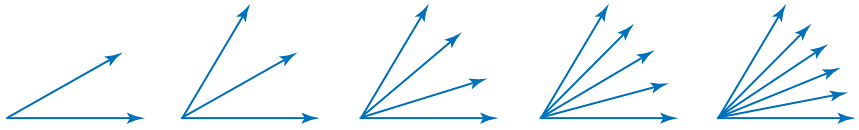
43. PHYSICS A ripple tank can be used to study the behavior of waves in two dimensions. As a wave strikes a barrier, it is reflected. The angle of incidence and the angle of reflection are congruent. In the diagram at the right, if $m\angle IBR = 62$, find the angle of reflection and $m\angle IBA$.



44. CRITICAL THINKING How would you compare the size of $\angle P$ and $\angle Q$? Explain.



CRITICAL THINKING **For Exercises 45–48, use the following information.**
Each figure below shows noncollinear rays with a common endpoint.



45. Count the number of angles in each figure.

46. Describe the pattern between the number of rays and the number of angles.

47. **Make a conjecture** of the number of angles that are formed by 7 noncollinear rays and by 10 noncollinear rays.

48. Write a formula for the number of angles formed by n noncollinear rays with a common endpoint.

49. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson.

How big is a degree?

Include the following in your answer:

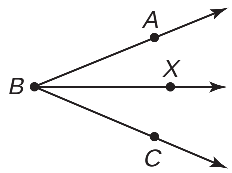
- how to find degree measure with a protractor, and
- drawings of several angles and their degree measures.

Standardized Test Practice

50. If $\overrightarrow{BX}$ bisects $\angle ABC$, which of the following are true?

Ⓐ $m\angle ABX = m\angle XBC$ Ⓑ $m\angle ABX = \frac{1}{2}m\angle ABC$

Ⓒ $\frac{1}{2}m\angle ABC = m\angle XBC$ Ⓓ all of these



51. ALGEBRA Solve $5n + 4 = 7(n + 1) - 2n$.

Ⓐ 0 Ⓑ −1 Ⓒ no solution Ⓓ all numbers

Maintain Your Skills

Mixed Review

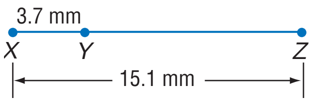
Find the distance between each pair of points. Then find the coordinates of the midpoint of the line segment between the points. *(Lesson 1-3)*

52. $A(2, 3)$, $B(5, 7)$ **53.** $C(-2, 0)$, $D(6, 4)$ **54.** $E(-3, -2)$, $F(5, 8)$

Find the measurement of each segment. *(Lesson 1-2)*

55. $\overline{WX}$

56. $\overline{YZ}$



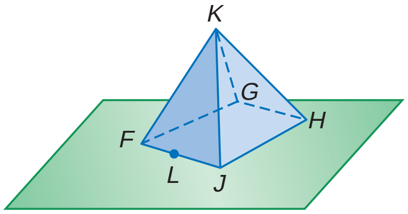
57. Find PQ if Q lies between P and R, $PQ = 6x - 5$, $QR = 2x + 7$, and $PQ = QR$. *(Lesson 1-2)*

Refer to the figure at the right. *(Lesson 1-1)*

58. How many planes are shown?

59. Name three collinear points.

60. Name a point coplanar with J, H, and F.



Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation.**

(To review ***solving equations****, see pages 737 and 738.)*

61. $14x + (6x - 10) = 90$ **62.** $2k + 30 = 180$

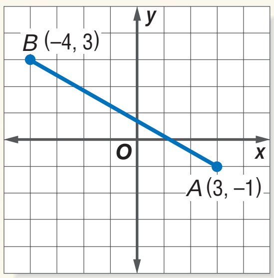
63. $180 - 5y = 90 - 7y$ **64.** $90 - 4t = \frac{1}{4}(180 - t)$

65. $(6m + 8) + (3m + 10) = 90$ **66.** $(7n - 9) + (5n + 45) = 180$

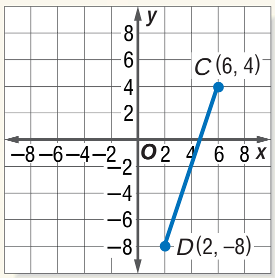
Practice Quiz 2 — *Lessons 1-3 and 1-4*

Find the coordinates of the midpoint of each segment. Then find the distance between the endpoints. *(Lesson 1-3)*

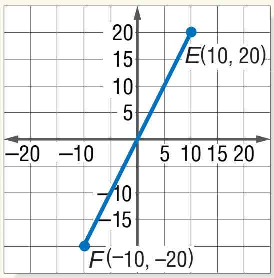
1.



2.



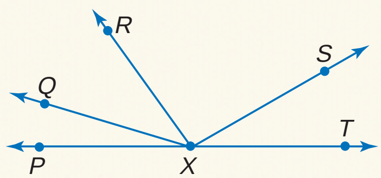
3.



In the figure, $\overrightarrow{XP}$ and $\overrightarrow{XT}$ are opposite rays. Given the following conditions, find the value of *a* and the measure of the indicated angle. *(Lesson 1-4)*

4. $m\angle SXT = 3a - 4$, $m\angle RXS = 2a + 5$, $m\angle RXT = 111$; $m\angle RXS$

5. $m\angle QXR = a + 10$, $m\angle QXS = 4a - 1$, $m\angle RXS = 91$; $m\angle QXS$



1-5 Angle Relationships

What You'll Learn

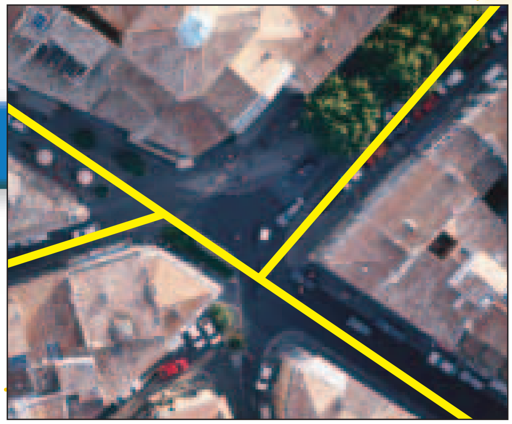
- Identify and use special pairs of angles.
- Identify perpendicular lines.

Vocabulary

- adjacent angles
- vertical angles
- linear pair
- complementary angles
- supplementary angles
- perpendicular

What kinds of angles are formed when streets intersect?

When two lines intersect, four angles are formed. In some cities, more than two streets might intersect to form even more angles. All of these angles are related in special ways.

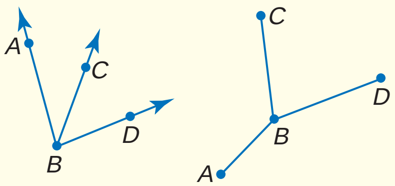


PAIRS OF ANGLES Certain pairs of angles have special names.

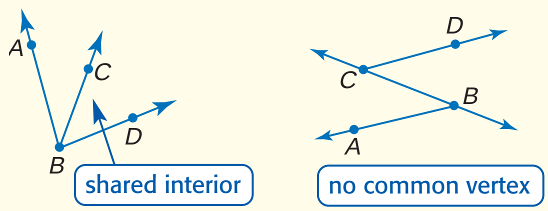
Key Concept — Angle Pairs

- **Words** **Adjacent angles** are two angles that lie in the same plane, have a common vertex, and a common side, but no common interior points.

- **Examples**
$\angle ABC$ and $\angle CBD$

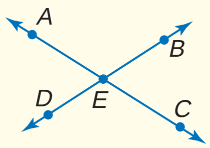


- **Nonexamples**
$\angle ABC$ and $\angle ABD$ $\angle ABC$ and $\angle BCD$

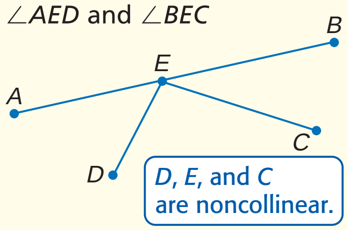


- **Words** **Vertical angles** are two nonadjacent angles formed by two intersecting lines.

- **Examples**
$\angle AEB$ and $\angle CED$
$\angle AED$ and $\angle BEC$

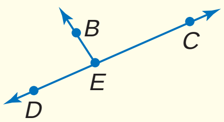


- **Nonexample**
$\angle AED$ and $\angle BEC$

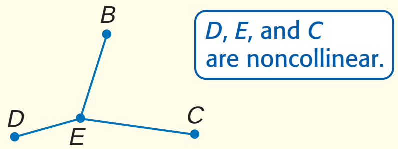


- **Words** A **linear pair** is a pair of adjacent angles whose noncommon sides are opposite rays.

- **Example**
$\angle BED$ and $\angle BEC$



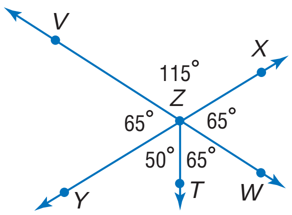
- **Nonexample**



Jason Hawkes/CORBIS

Example 1 Identify Angle Pairs

Name an angle pair that satisfies each condition.



a. two obtuse vertical angles

$\angle VZX$ and $\angle YZW$ are vertical angles.

They each have measures greater than 90°, so they are obtuse.

b. two acute adjacent angles

There are four acute angles shown.

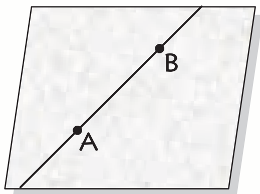
Adjacent acute angles are $\angle VZY$ and $\angle YZT$, $\angle YZT$ and $\angle TZW$, and $\angle TZW$ and $\angle WZX$.

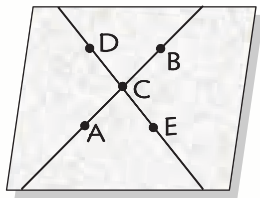
The measures of angles formed by intersecting lines also have a special relationship.

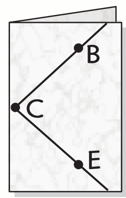
Geometry Activity

Angle Relationships

Make a Model







Step 1

Fold a piece of patty paper so that it makes a crease across the paper. Open the paper, trace the crease with a pencil, and name two points on the crease *A* and *B*.

Step 2

Fold the paper again so that the crease intersects $\overleftrightarrow{AB}$ between the two labeled points. Open the paper, trace this crease, and label the intersection *C*. Label two other points, *D* and *E*, on the second crease so that *C* is between *D* and *E*.

Step 3

Fold the paper again through point *C* so that $\overrightarrow{CB}$ aligns with $\overrightarrow{CD}$.

Analyze the Model

1. What do you notice about $\angle BCE$ and $\angle DCA$ when you made the last fold?
2. Fold the paper again through *C* so that $\overrightarrow{CB}$ aligns with $\overrightarrow{CE}$. What do you notice?
3. Use a protractor to measure each angle. Label the measure on your model.
4. Name pairs of vertical angles and their measures.
5. Name linear pairs of angles and their measures.
6. Compare your results with those of your classmates. Write a "rule" about the measures of vertical angles and another about the measures of linear pairs.

Study Tip

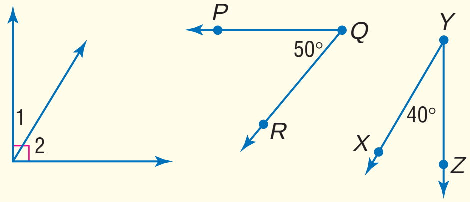
Patty Paper
Patty paper is the squares of paper used to separate hamburger patties. You can also use waxed paper or tracing paper.

The Geometry Activity suggests that all vertical angles are congruent. It also supports the concept that the sum of the measures of a linear pair is 180.

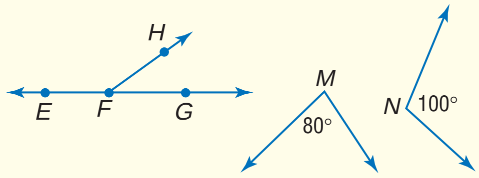
There are other angle relationships that you may remember from previous math courses. These are complementary angles and supplementary angles.

Key Concept — Angle Relationships

- **Words** **Complementary angles** are two angles whose measures have a sum of 90.
- **Examples**
  $\angle 1$ and $\angle 2$ are complementary.
  $\angle PQR$ and $\angle XYZ$ are complementary.



- **Words** **Supplementary angles** are two angles whose measures have a sum of 180.
- **Examples**
  $\angle EFH$ and $\angle HFG$ are supplementary.
  $\angle M$ and $\angle N$ are supplementary.



Study Tip

Complementary and Supplementary Angles

While the other angle pairs in this lesson share at least one point, complementary and supplementary angles need not share any points.

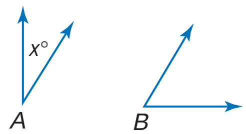
Remember that angle measures are real numbers. So, the operations for real numbers and algebra can be used with angle measures.

Example 2 Angle Measure

ALGEBRA **Find the measures of two complementary angles if the difference in the measures of the two angles is 12.**

Explore The problem relates the measures of two complementary angles. You know that the sum of the measures of complementary angles is 90.

Plan Draw two figures to represent the angles.



Let the measure of one angle be x.
If $m\angle A = x$, then because $\angle A$ and $\angle B$ are complementary, $m\angle B + x = 90$ or $m\angle B = 90 - x$.

The problem states that the difference of the two angle measures is 12, or $m\angle B - m\angle A = 12$.

Solve

$m\angle B - m\angle A = 12$	Given
$(90 - x) - x = 12$	$m\angle A = x$, $m\angle B = 90 - x$
$90 - 2x = 12$	Simplify.
$-2x = -78$	Subtract 90 from each side.
$x = 39$	Divide each side by -2.

Use the value of x to find each angle measure.

$m\angle A = x$ $\quad$ $m\angle B = 90 - x$

$m\angle A = 39$ $\quad$ $m\angle B = 90 - 39$ or 51

Examine Add the angle measures to verify that the angles are complementary.

$$m\angle A + m\angle B = 90$$
$$39 + 51 = 90$$
$$90 = 90$$

www.geometryonline.com/extra_examples

CONTENTS

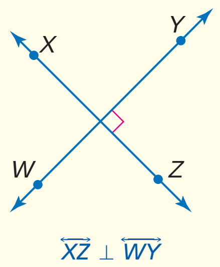
PERPENDICULAR LINES

Lines that form right angles are **perpendicular**. The following statements are also true when two lines are perpendicular.

> ### Study Tip
> **Interpreting Figures**
> Never assume that two lines are perpendicular because they appear to be so in the figure. The only sure way to know if they are perpendicular is if the right angle symbol is present or if the problem states angle measures that allow you to make that conclusion.

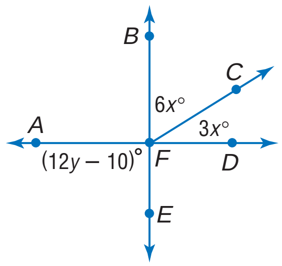
Key Concept — Perpendicular Lines

- Perpendicular lines intersect to form four right angles.
- Perpendicular lines intersect to form congruent adjacent angles.
- Segments and rays can be perpendicular to lines or to other line segments and rays.
- The right angle symbol in the figure indicates that the lines are perpendicular.
- **Symbol** $\perp$ is read *is perpendicular to.*



Example 3 Perpendicular Lines

ALGEBRA **Find x and y so that $\overleftrightarrow{BE}$ and $\overleftrightarrow{AD}$ are perpendicular.**



If $\overleftrightarrow{BE} \perp \overleftrightarrow{AD}$, then $m\angle BFD = 90$ and $m\angle AFE = 90$.

To find x, use $\angle BFC$ and $\angle CFD$.

$m\angle BFD = m\angle BFC + m\angle CFD$	Sum of parts = whole
$90 = 6x + 3x$	Substitution
$90 = 9x$	Add.
$10 = x$	Divide each side by 9.

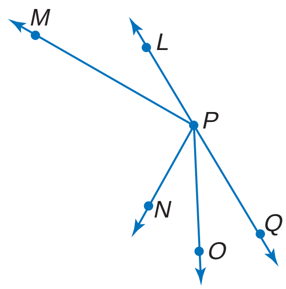
To find y, use $\angle AFE$.

$m\angle AFE = 12y - 10$	Given
$90 = 12y - 10$	Substitution
$100 = 12y$	Add 10 to each side.
$\frac{25}{3} = y$	Divide each side by 12, and simplify.

> ### Study Tip
> **Naming Figures**
> The list of statements that can be assumed is not a complete list. There are more special pairs of angles than those listed. Also remember that all figures except points usually have more than one way to name them.

While two lines may appear to be perpendicular in a figure, you cannot assume this is true unless other information is given. In geometry, figures are used to depict a situation. They are not drawn to reflect total accuracy of the situation. There are certain relationships you can assume to be true, but others that you cannot.

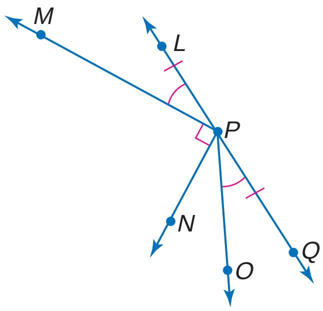
Study the figure at the right and then compare the lists below.



Can Be Assumed	Cannot Be Assumed
All points shown are coplanar.	Perpendicular lines: $\overleftrightarrow{PN} \perp \overleftrightarrow{PM}$
L, P, and Q are collinear.	Congruent angles: $\angle QPO \cong \angle LPM$
$\overleftrightarrow{PM}$, $\overleftrightarrow{PN}$, $\overleftrightarrow{PO}$, and $\overleftrightarrow{LQ}$ intersect at P.	$\angle QPO \cong \angle OPN$
P is between L and Q.	$\angle OPN \cong \angle LPM$
N is in the interior of MPO.	Congruent segments: $\overline{LP} \cong \overline{PQ}$
$\angle LPM$ and $\angle MPN$ are adjacent angles.	$\overline{PQ} \cong \overline{PO}$
$\angle LPN$ and $\angle NPQ$ are a linear pair.	$\overline{PO} \cong \overline{PN}$
$\angle QPO$ and $\angle OPL$ are supplementary.	$\overline{PN} \cong \overline{PL}$

Example 4 Interpret Figures

Determine whether each statement can be assumed from the figure below.



a. ∠*LPM* and ∠*MPO* are adjacent angles.

Yes; they share a common side and vertex and have no interior points in common.

b. ∠*OPQ* and ∠*LPM* are complementary.

No; they are congruent, but we do not know anything about their exact measures.

c. ∠*LPO* and ∠*QPO* are a linear pair.

Yes; they are adjacent angles whose noncommon sides are opposite rays.

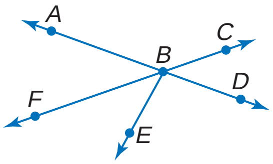
Check for Understanding

Concept Check

1. **OPEN ENDED** Draw two angles that are supplementary, but not adjacent.
2. **Explain** the statement *If two adjacent angles form a linear pair, they must be supplementary.*
3. **Write** a sentence to explain why a linear pair of angles is called *linear*.

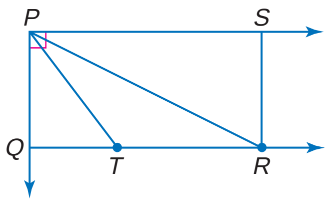
Guided Practice

For Exercises 4 and 5, use the figure at the right and a protractor.



4. Name two acute vertical angles.
5. Name two obtuse adjacent angles.
6. The measure of the supplement of an angle is 60 less than three times the measure of the complement of the angle. Find the measure of the angle.
7. Lines p and q intersect to form adjacent angles 1 and 2. If $m\angle 1 = 3x + 18$ and $m\angle 2 = -8y - 70$, find the values of x and y so that p is perpendicular to q.

Determine whether each statement can be assumed from the figure. Explain.



8. ∠*SRP* and ∠*PRT* are complementary.
9. ∠*QPT* and ∠*TPR* are adjacent, but neither complementary or supplementary.

Application

10. **SKIING** Alisa Camplin won the gold medal in the 2002 Winter Olympics with a triple-twisting, double backflip jump in the women's freestyle skiing event. While she is in the air, her skis are positioned like intersecting lines. If ∠4 measures 60°, find the measures of the other angles.

Reuters New Media/CORBIS

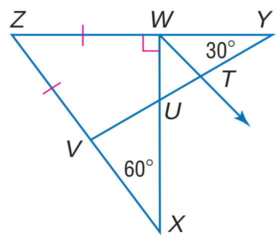
Practice and Apply

Homework Help

For Exercises	See Examples
11–16	1
17–22	2
27–30	3
31–35	4

Extra Practice
See page 755.

For Exercises 11–16, use the figure at the right and a protractor.



11. Name two acute vertical angles.

12. Name two obtuse vertical angles.

13. Name a pair of complementary adjacent angles.

14. Name a pair of complementary nonadjacent angles.

15. Name a linear pair whose vertex is T.

16. Name an angle supplementary to $\angle UVZ$.

17. Rays PQ and QR are perpendicular. Point S lies in the interior of $\angle PQR$. If $m\angle PQS = 4 + 7a$ and $m\angle SQR = 9 + 4a$, find $m\angle PQS$ and $m\angle SQR$.

18. The measures of two complementary angles are $16z - 9$ and $4z + 3$. Find the measures of the angles.

19. Find $m\angle T$ if $m\angle T$ is 20 more than four times its supplement.

20. The measure of an angle's supplement is 44 less than the measure of the angle. Find the measure of the angle and its supplement.

21. Two angles are supplementary. One angle measures 12° more than the other. Find the measures of the angles.

22. The measure of $\angle 1$ is five less than four times the measure of $\angle 2$. If $\angle 1$ and $\angle 2$ form a linear pair, what are their measures?

Determine whether each statement is *sometimes*, *always*, or *never* true.

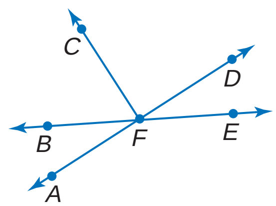
23. If two angles are supplementary and one is acute, the other is obtuse.

24. If two angles are complementary, they are both acute angles.

25. If $\angle A$ is supplementary to $\angle B$ and $\angle B$ is supplementary to $\angle C$, then $\angle A$ is supplementary to $\angle C$.

26. If $\overline{PN} \perp \overline{PQ}$, then $\angle NPQ$ is acute.

ALGEBRA **For Exercises 27–29, use the figure at the right.**



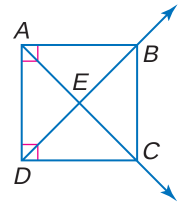
27. If $m\angle CFD = 12a + 45$, find a so that $\overrightarrow{FC} \perp \overrightarrow{FD}$.

28. If $m\angle AFB = 8x - 6$ and $m\angle BFC = 14x + 8$, find the value of x so that $\angle AFC$ is a right angle.

29. If $\angle BFA = 3r + 12$ and $m\angle DFE = -8r + 210$, find $m\angle AFE$.

30. $\angle L$ and $\angle M$ are complementary angles. $\angle N$ and $\angle P$ are complementary angles. If $m\angle L = y - 2$, $m\angle M = 2x + 3$, $m\angle N = 2x - y$, and $m\angle P = x - 1$, find the values of x, y, $m\angle L$, $m\angle M$, $m\angle N$, and $m\angle P$.

Determine whether each statement can be assumed from the figure. Explain.



31. $\angle DAB$ is a right angle.

32. $\angle AEB \cong \angle DEC$

33. $\angle ADB$ and $\angle BDC$ are complementary.

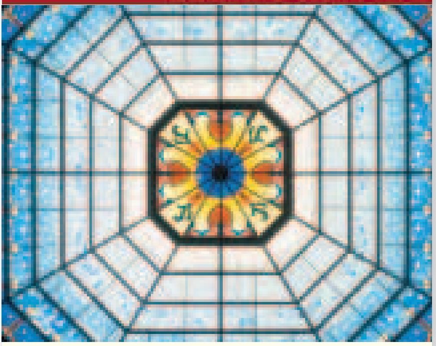
34. $\angle DAE \cong \angle ADE$

35. $\overline{AB} \perp \overline{BC}$

36. **LANGUAGE** Look up the words *complementary* and *complimentary*. Discuss the differences and which has a mathematical meaning.

37. **CRITICAL THINKING** A counterexample is used to show that a statement is not necessarily true. Find a counterexample for the statement *Supplementary angles form linear pairs.*

More About. . .

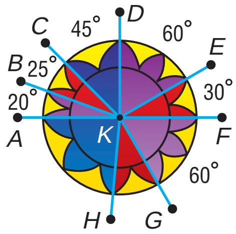


Stained Glass

In the 13th century, artists began using geometric patterns and shapes to create complex, colorful windows.

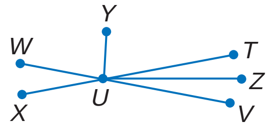
Source: www.infoplease.com

38. STAINED GLASS Stained glass patterns often have several pieces of glass held together by lead at a central point. In the pattern at the right, determine which segments are perpendicular.



39. CRITICAL THINKING In the figure, $\angle WUT$ and $\angle XUV$ are vertical angles, $\overline{YU}$ is the bisector of $\angle WUT$, and $\overline{UZ}$ is the bisector of $\angle TUV$. Write a convincing argument that $\overline{YU} \perp \overline{UZ}$.

40. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.



What kinds of angles are formed when streets intersect?

Include the following in your answer.

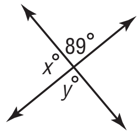
- the types of angles that might be formed by two intersecting lines, and
- a sketch of intersecting streets with angle measures marked and all angle pairs identified.

Standardized Test Practice

41. Which statement is true of the figure?

Ⓐ $x > y$ Ⓑ $x < y$

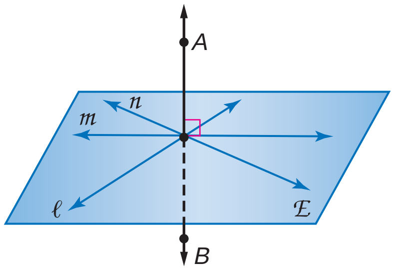
Ⓒ $x = y$ Ⓓ cannot be determined



42. SHORT RESPONSE The product of 4, 5, and 6 is equal to twice the sum of 10 and what number?

Extending the Lesson

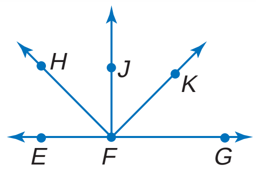
43. The concept of perpendicularity can be extended to include planes. If a line, line segment, or ray is perpendicular to a plane, it is perpendicular to every line, line segment, or ray in that plane that intersects it. In the figure at the right, $\overleftrightarrow{AB} \perp \mathcal{E}$. Name all pairs of perpendicular lines.



Maintain Your Skills

Mixed Review

Measure each angle and classify it as *right*, *acute*, or *obtuse*. *(Lesson 1-4)*



44. $\angle KFG$ **45.** $\angle HFG$

46. $\angle HFK$ **47.** $\angle JFE$

48. $\angle HFJ$ **49.** $\angle EFK$

Find the distance between each pair of points. *(Lesson 1-3)*

50. $A(3, 5), B(0, 1)$ **51.** $C(5, 1), D(5, 9)$ **52.** $E(-2, -10), F(-4, 10)$

53. $G(7, 2), H(-6, 0)$ **54.** $J(-8, 9), K(4, 7)$ **55.** $L(1, 3), M(3, -1)$

Find the value of the variable and *QR* if *Q* is between *P* and *R*. *(Lesson 1-2)*

56. $PQ = 1 - x, QR = 4x + 17, PR = -3x$

57. $PR = 7n + 8, PQ = 4n - 3, QR = 6n + 2$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression if $\ell = 3$, $w = 8$, and $s = 2$.**

*(To review **evaluating expressions**, see page 736.)*

58. $2\ell + 2w$ **59.** ℓw **60.** $4s$ **61.** $\ell w + ws$ **62.** $s(\ell + w)$

Geometry Activity

A Follow-Up of Lesson 1-5

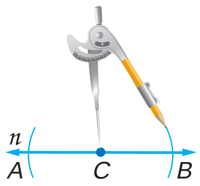
Constructing Perpendiculars

You can use a compass and a straightedge to construct a line perpendicular to a given line through a point on the line, or through a point *not* on the line.

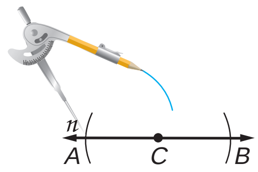
Activity 1 Perpendicular Through a Point on the Line

Construct a line perpendicular to line n and passing through point C on n.

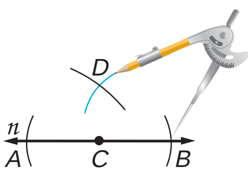
1. Place the compass at point C. Using the same compass setting, draw arcs to the right and left of C, intersecting line n. Label the points of intersection A and B.



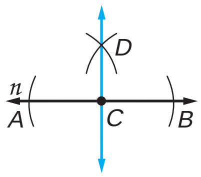
2. Open the compass to a setting greater than AC. Put the compass at point A and draw an arc above line n.



3. Using the same compass setting as in Step 2, place the compass at point B and draw an arc intersecting the arc drawn in Step 2. Label the point of intersection D.



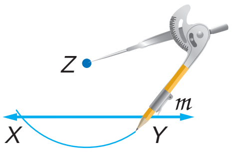
4. Use a straightedge to draw $\overleftrightarrow{CD}$.



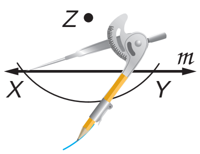
Activity 2 Perpendicular Through a Point not on the Line

Construct a line perpendicular to line m and passing through point Z not on m.

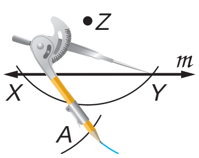
1. Place the compass at point Z. Draw an arc that intersects line m in two different places. Label the points of intersection X and Y.



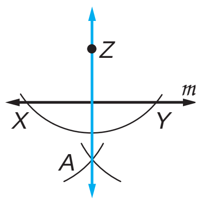
2. Open the compass to a setting greater than $\frac{1}{2}XY$. Put the compass at point X and draw an arc below line m.



3. Using the same compass setting, place the compass at point Y and draw an arc intersecting the arc drawn in Step 2. Label the point of intersection A.



4. Use a straightedge to draw $\overleftrightarrow{ZA}$.



Model and Analyze

1. Draw a line and construct a line perpendicular to it through a point on the line. Repeat with a point not on the line.
2. How is the second construction similar to the first one?

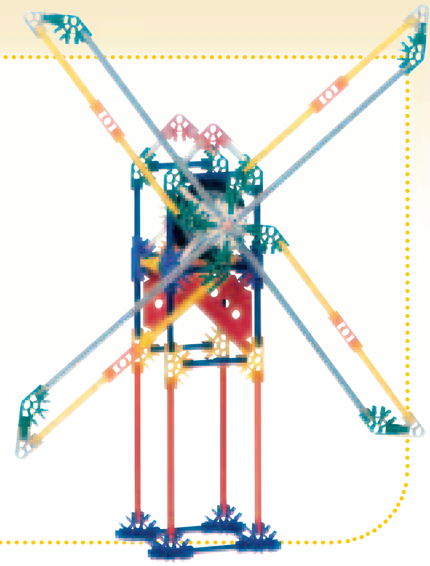
1-6 Polygons

What You'll Learn

- Identify and name polygons.
- Find perimeters of polygons.

How are polygons related to toys?

There are numerous types of building sets that connect sticks to form various shapes. Whether they are made of plastic, wood, or metal, the sticks represent segments. When the segments are connected, they form angles. The sticks are connected to form closed figures that in turn are connected to make a model of a real-world object.



Vocabulary

- polygon
- concave
- convex
- *n*-gon
- regular polygon
- perimeter

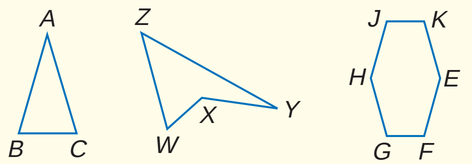
Study Tip

Reading Math
The plural of vertex is *vertices.*

POLYGONS Each closed figure shown in the toy is a **polygon**. A polygon is a closed figure whose sides are all segments. The sides of each angle in a polygon are called *sides* of the polygon, and the vertex of each angle is a *vertex* of the polygon.

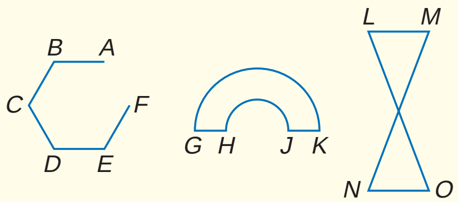
Key Concept *Polygon*

- **Words** A polygon is a closed figure formed by a finite number of coplanar segments such that
  (1) the sides that have a common endpoint are noncollinear, and
  (2) each side intersects exactly two other sides, but only at their endpoints.
- **Symbol** A polygon is named by the letters of its vertices, written in consecutive order.
- **Examples**

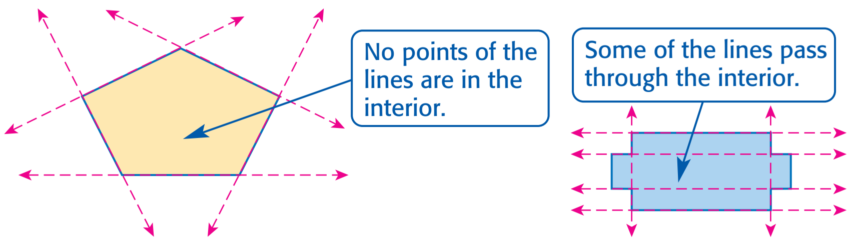


polygons *ABC*, *WXYZ*, *EFGHJK*

- **Nonexamples**



Polygons can be **concave** or **convex**. Suppose the line containing each side is drawn. If any of the lines contain any point in the interior of the polygon, then it is concave. Otherwise it is convex.



convex polygon

concave polygon

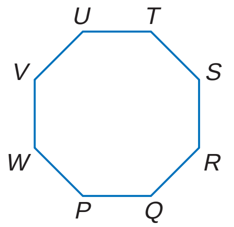
Study Tip

Reading Math
The term *polygon* is derived from a Greek word meaning *many angles*. Since *hexa-* means 6, you would think *hexagon* means 6 angles, and you would be correct. Every polygon has the same number of angles as it does sides.

You are already familiar with many polygon names, such as triangle, square, and rectangle. In general, polygons can be classified by the number of sides they have. A polygon with n sides is an **n-gon**. The table lists some common names for various categories of polygon.

Number of Sides	Polygon
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
12	dodecagon
n	n-gon

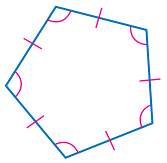
A convex polygon in which all the sides are congruent and all the angles are congruent is called a **regular polygon**. Octagon $PQRSTUVW$ below is a regular octagon.



Example 1 Identify Polygons

Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

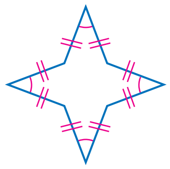
a.



There are 5 sides, so this is a pentagon. No line containing any of the sides will pass through the interior of the pentagon, so it is convex.

The sides are congruent, and the angles are congruent. It is regular.

b.



There are 8 sides, so this is an octagon. A line containing any of the sides will pass through the interior of the octagon, so it is concave.

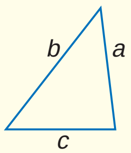
The sides are congruent. However, since it is concave, it cannot be regular.

PERIMETER The **perimeter** of a polygon is the sum of the lengths of its sides, which are segments. Some shapes have special formulas, but they are all derived from the basic definition of perimeter.

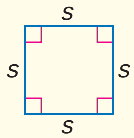
Key Concept — Perimeter

- **Words** The perimeter P of a polygon is the sum of the lengths of the sides of a polygon.
- **Examples**

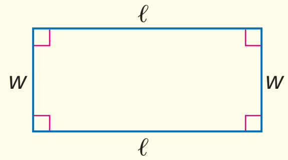
triangle
$P = a + b + c$



square
$P = s + s + s + s$
$P = 4s$



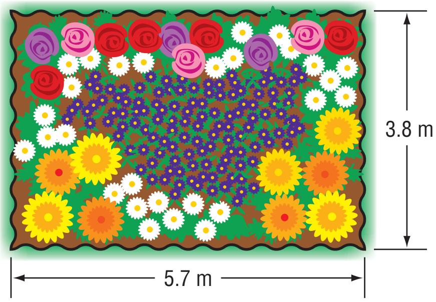
rectangle
$P = \ell + w + \ell + w$
$P = 2\ell + 2w$



Example 2 Find Perimeter

GARDENING **A landscape designer is putting black plastic edging around a rectangular flower garden that has length 5.7 meters and width 3.8 meters. The edging is sold in 5-meter lengths.**

a. Find the perimeter of the garden and determine how much edging the designer should buy.



$P = 2\ell + 2w$

$= 2(5.7) + 2(3.8)$ $\ell = 5.7, w = 3.8$

$= 11.4 + 7.6$ or 19

The perimeter of the garden is 19 meters.

The designer needs to buy 20 meters of edging.

b. Suppose the length and width of the garden are tripled. What is the effect on the perimeter and how much edging should the designer buy?

The new length would be 3(5.7) or 17.1 meters.

The new width would be 3(3.8) or 11.4 meters.

$P = 2\ell + 2w$

$= 2(17.1) + 2(11.4)$ or 57

Compare the original perimeter to this measurement.

$57 = 3(19)$ meters

So, when the lengths of the sides of the rectangle are tripled, the perimeter also triples. The designer needs to buy 60 meters of edging.

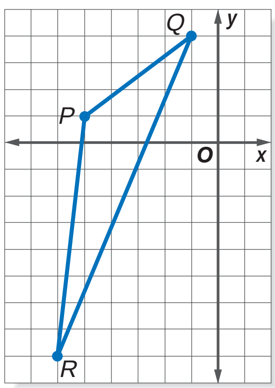
You can use the Distance Formula to find the perimeter of a polygon graphed on a coordinate plane.

Example 3 Perimeter on the Coordinate Plane

Study Tip

Look Back
To review the **Distance Formula**, see Lesson 1-3.

COORDINATE GEOMETRY **Find the perimeter of triangle PQR if $P(-5, 1)$, $Q(-1, 4)$, and $R(-6, -8)$.**



Use the Distance Formula,

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$,

to find PQ, QR, and PR.

$PQ = \sqrt{[-1 - (-5)]^2 + (4 - 1)^2}$

$= \sqrt{4^2 + 3^2}$

$= \sqrt{25}$ or 5

$QR = \sqrt{[-6 - (-1)]^2 + (-8 - 4)^2}$

$= \sqrt{(-5)^2 + (-12)^2}$

$= \sqrt{169}$ or 13

$PR = \sqrt{[-6 - (-5)]^2 + (-8 - 1)^2}$

$= \sqrt{(-1)^2 + (-9)^2}$

$= \sqrt{82} \approx 9.1$

The perimeter of triangle PQR is $5 + 13 + \sqrt{82}$ or about 27.1 units.

You can also use algebra to find the lengths of the sides if the perimeter is known.

Study Tip

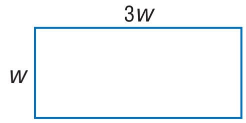
Equivalent Measures

In Example 5, the dimensions $\frac{1}{4}$ foot by $\frac{3}{4}$ foot can also be expressed as 3 inches by 9 inches.

Example 4 Use Perimeter to Find Sides

ALGEBRA The length of a rectangle is three times the width. The perimeter is 2 feet. Find the length of each side.

Let w represent the width. Then the length is $3w$.



$P = 2\ell + 2w$	Perimeter formula for rectangle
$2 = 2(3w) + 2w$	$\ell = 3w$
$2 = 8w$	Simplify.
$\frac{1}{4} = w$	Divide each side by 8.

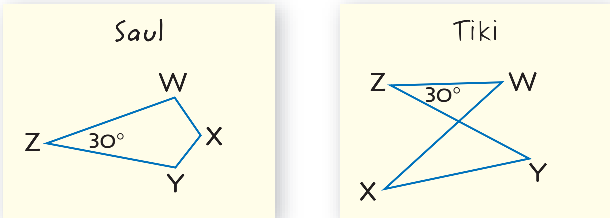
The width is $\frac{1}{4}$ foot. By substituting $\frac{1}{4}$ for w, the length $3w$ becomes $3\left(\frac{1}{4}\right)$ or $\frac{3}{4}$ foot.

Check for Understanding

Concept Check

1. **OPEN ENDED** Explain how you would find the length of a side of a regular decagon if the perimeter is 120 centimeters.

2. **FIND THE ERROR** Saul and Tiki were asked to draw quadrilateral $WXYZ$ with $m\angle Z = 30$.



Who is correct? Explain your reasoning.

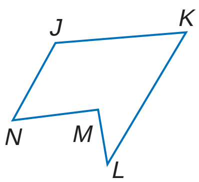
3. **Write** a formula for the perimeter of a triangle with congruent sides of length s.

4. **Draw** a concave pentagon and explain why it is concave.

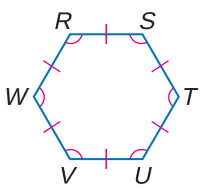
Guided Practice

Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

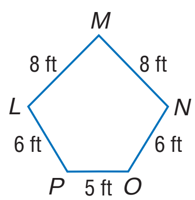
5.



6.



For Exercises 7 and 8, use pentagon *LMNOP*.



7. Find the perimeter of pentagon $LMNOP$.

8. Suppose the length of each side of pentagon $LMNOP$ is doubled. What effect does this have on the perimeter?

9. **COORDINATE GEOMETRY** A polygon has vertices $P(-3, 4)$, $Q(0, 8)$, $R(3, 8)$, and $S(0, 4)$. Find the perimeter of $PQRS$.

10. **ALGEBRA** Quadrilateral $ABCD$ has a perimeter of 95 centimeters. Find the length of each side if $AB = 3a + 2$, $BC = 2(a - 1)$, $CD = 6a + 4$, and $AD = 5a - 5$.

Application

11. **HISTORIC LANDMARKS** The Pentagon building in Arlington, Virginia, is so named because of its five congruent sides. Find the perimeter of the outside of the Pentagon if one side is 921 feet long.

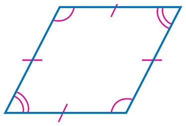
Practice and Apply

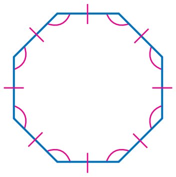
Homework Help

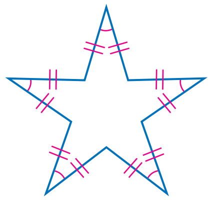
For Exercises	See Examples
12–18	1
19–25	2
26–28	3
29–34	4

Extra Practice
See page 755.

Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

12. 

13. 

14. 

TRAFFIC SIGNS **Identify the shape of each traffic sign.**

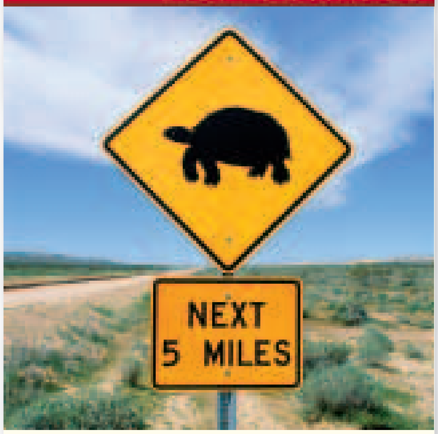
15. school zone


16. caution or warning


17. yield


18. railroad


More About...

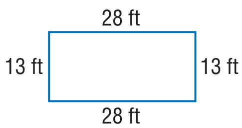


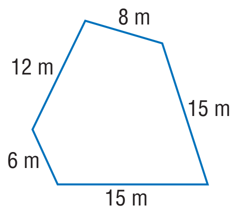
Traffic Signs

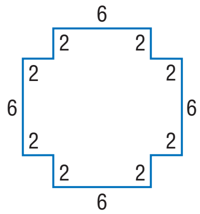
The shape of a traffic sign is determined by the use of the sign. For example, tall rectangle shaped signs are used for posting speed limits, parking restrictions, and other regulations. Wide rectangle shaped signs are used to provide drivers with information.

Source: U.S. Federal Highway Administration

Find the perimeter of each figure.

19. 

20. 

21. 

22. What is the effect on the perimeter of the figure in Exercise 19 if each measure is multiplied by 4?

23. What is the effect on the perimeter of the figure in Exercise 20 if each measure is tripled?

24. What is the effect on the perimeter of the figure in Exercise 21 if each measure is divided by 2?

25. The perimeter of an n-gon is 12.5 meters. Find the perimeter of the n-gon if the length of each of its n sides is multiplied by 10.

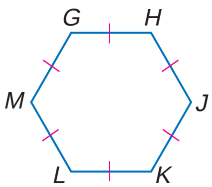
COORDINATE GEOMETRY **Find the perimeter of each polygon.**

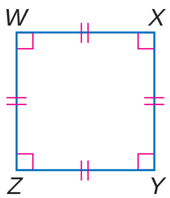
26. rectangle with vertices $A(-1, 1)$, $B(3, 4)$, $C(6, 0)$, and $D(2, -3)$

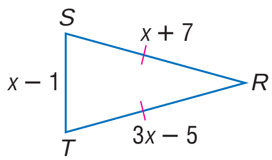
27. hexagon with vertices $P(-2, 3)$, $Q(3, 3)$, $R(7, 0)$, $S(3, -3)$, $T(-2, -3)$, and $U(-6, 0)$

28. pentagon with vertices $V(3, 0)$, $W(-2, 12)$, $X(-10, -3)$, $Y(-8, -12)$, and $Z(-2, -12)$

ALGEBRA **Find the length of each side of the polygon for the given perimeter.**

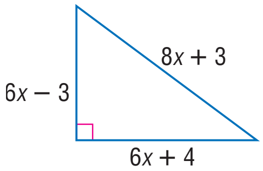
29. $P = 90$ centimeters


30. $P = 14$ miles


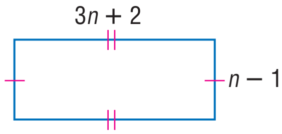
31. $P = 31$ units


ALGEBRA **Find the length of each side of the polygon for the given perimeter.**

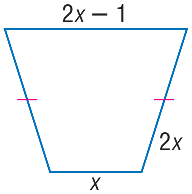
32. $P = 84$ meters



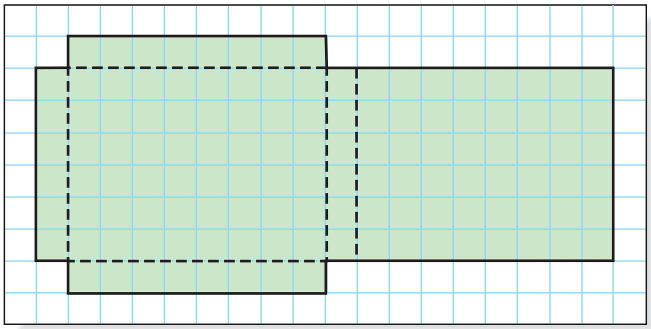
33. $P = 42$ inches



34. $P = 41$ yards



35. NETS *Nets* are patterns that form a three-dimensional figure when cut out and folded. The net at the right makes a rectangular box. What is the perimeter of the net?



36. CRITICAL THINKING Use grid paper to draw all possible rectangles with length and width that are whole numbers and with a perimeter of 12. Record the number of grid squares contained in each rectangle.

a. What do you notice about the rectangle with the greatest number of squares?

b. The perimeter of another rectangle is 36. What would be the dimensions of the rectangle with the greatest number of squares?

37. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are polygons related to toys?

Include the following in your answer:

- names of the polygons shown in the picture of the toy structure, and
- sketches of other polygons that could be formed with construction toys with which you are familiar.

Standardized Test Practice

38. SHORT RESPONSE A farmer fenced all but one side of a square field. If he has already used $3x$ meters of fence, how many meters will he need for the last side?

39. ALGEBRA If $5n + 5 = 10$, what is the value of $11 - n$?

Ⓐ -10 Ⓑ 0 Ⓒ 5 Ⓓ 10

Maintain Your Skills

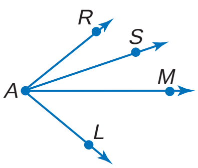
Mixed Review

Determine whether each statement is *always*, *sometimes*, or *never true*. *(Lesson 1-5)*

40. Two angles that form a linear pair are supplementary.

41. If two angles are supplementary, then one of the angles is obtuse.

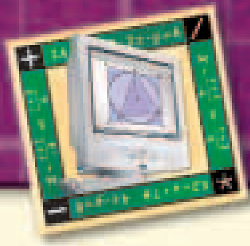
In the figure, $\overrightarrow{AM}$ bisects $\angle LAR$, and $\overrightarrow{AS}$ bisects $\angle MAR$. *(Lesson 1-4)*



42. If $m\angle MAR = 2x + 13$ and $m\angle MAL = 4x - 3$, find $m\angle RAL$.

43. If $m\angle RAL = x + 32$ and $m\angle MAR = x - 31$, find $m\angle LAM$.

44. Find $m\angle LAR$ if $m\angle RAS = 25 - 2x$ and $m\angle SAM = 3x + 5$.



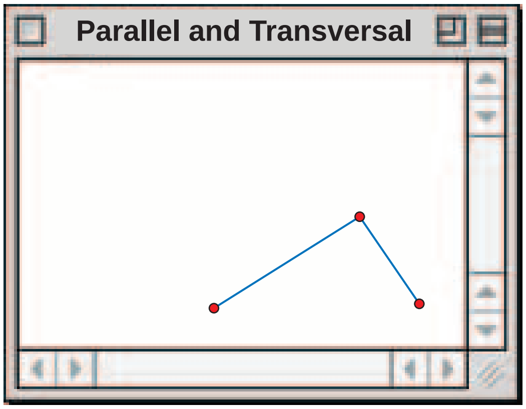
Geometry Software Investigation

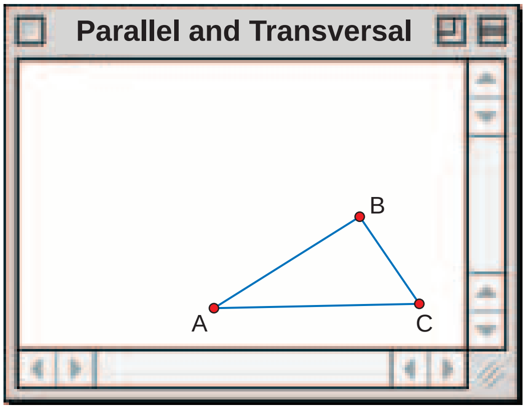
A Follow-Up of Lesson 1-6

Measuring Polygons

You can use The Geometer's Sketchpad® to draw and investigate polygons. It can be used to find the measures of the sides and the perimeter of a polygon. You can also find the measures of the angles in a polygon.

Step 1 Draw $\triangle ABC$.



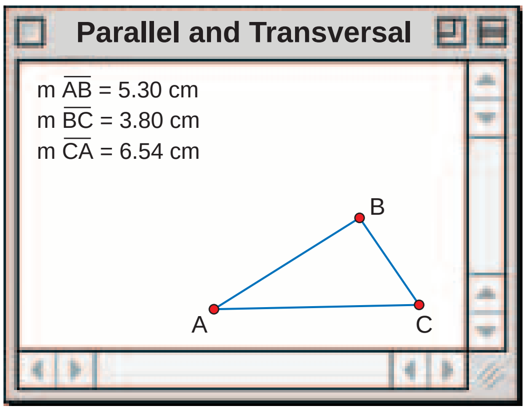


- Select the segment tool from the toolbar, and click to set the first endpoint A of side $\overline{AB}$. Then drag the cursor and click again to set the other endpoint B.
- Click on point B to set the endpoint of $\overline{BC}$. Drag the cursor and click to set point C.
- Click on point C to set the endpoint of $\overline{CA}$. Then move the cursor to highlight point A. Click on A to draw $\overline{CA}$.
- Use the pointer tool to click on points A, B, and C. Under the **Display** menu, select **Show Labels** to label the vertices of your triangle.

Step 2 Find AB, BC, and CA.

- Use the pointer tool to select $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.
- Select the **Length** command under the **Measure** menu to display the lengths of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.

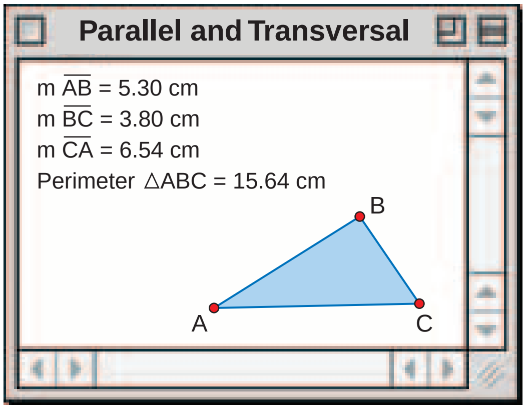
$AB = 5.30$
$BC = 3.80$
$CA = 6.54$



Geometry Software Investigation

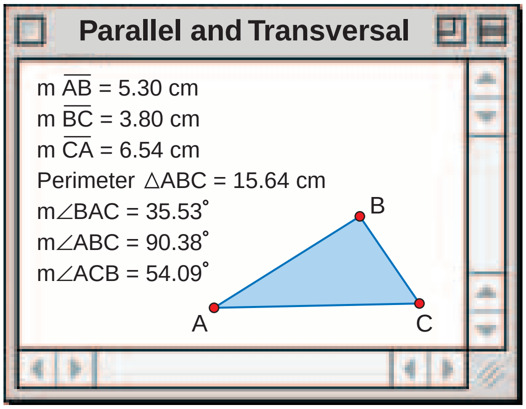
Step 3 *Find the perimeter of $\triangle ABC$.*

- Use the pointer tool to select points A, B, and C.
- Under the **Construct** menu, select **Triangle Interior**. The triangle will now be shaded.
- Select the triangle interior using the pointer.
- Choose the **Perimeter** command under the **Measure** menu to find the perimeter of $\triangle ABC$. The perimeter of $\triangle ABC$ is 15.64 centimeters.



Step 4 *Find $m\angle A$, $m\angle B$, and $m\angle C$.*

- Recall that $\angle A$ can also be named $\angle BAC$ or $\angle CAB$. Use the pointer to select points B, A, and C in order.
- Select the **Angle** command from the **Measure** menu to find $m\angle A$.
- Select points A, B, and C. Find $m\angle B$.
- Select points A, C, and B. Find $m\angle C$.



Analyze

1. Add the side measures you found in Step 2. Compare this sum to the result of Step 3. How do these compare?

2. What is the sum of the angle measures of $\triangle ABC$?

3. Repeat the activities for each convex polygon.

a. irregular quadrilateral **b.** square **c.** pentagon **d.** hexagon

4. Draw another quadrilateral and find its perimeter. Then enlarge your figure using the **Dilate** command under the **Transform** menu. How does changing the sides affect the perimeter?

5. Compare your results with those of your classmates.

Make a Conjecture

6. Make a conjecture about the sum of the measures of the angles in any triangle.

7. What is the sum of the measures of the angles of a quadrilateral? pentagon? hexagon?

8. Make a conjecture about how the sums of the measures of the angles of polygons are related to the number of sides.

9. Test your conjecture on other polygons. Does your conjecture hold for these polygons? Explain.

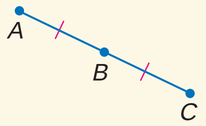
10. When the sides of a polygon are changed by a common factor, does the perimeter of the polygon change by the same factor as the sides? Explain.

Chapter 1 Study Guide and Review

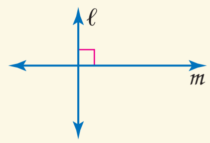
Vocabulary and Concept Check

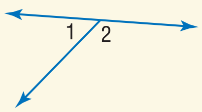
- acute angle (p. 30)
- adjacent angles (p. 37)
- angle (p. 29)
- angle bisector (p. 32)
- between (p. 14)
- betweenness of points (p. 14)
- collinear (p. 6)
- complementary angles (p. 39)
- concave (p. 45)
- congruent (p. 15)
- construction (p. 15)
- convex (p. 45)
- coplanar (p. 6)
- degree (p. 29)
- exterior (p. 29)
- interior (p. 29)
- line (p. 6)
- line segment (p. 13)
- linear pair (p. 37)
- locus (p. 11)
- midpoint (p. 22)
- *n*-gon (p. 46)
- obtuse angle (p. 30)
- opposite rays (p. 29)
- perimeter (p. 46)
- perpendicular (p. 40)
- plane (p. 6)
- point (p. 6)
- polygon (p. 45)
- precision (p. 14)
- ray (p. 29)
- regular polygon (p. 46)
- relative error (p. 19)
- right angle (p. 30)
- segment bisector (p. 24)
- sides (p. 29)
- space (p. 8)
- supplementary angles (p. 39)
- undefined terms (p. 7)
- vertex (p. 29)
- vertical angles (p. 37)

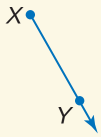
Exercises **Choose the letter of the term that best matches each figure.**

1. 

2. 

3. 

4. 

5. 

6. 

a. line
b. ray
c. complementary angles
d. midpoint
e. supplementary angles
f. perpendicular
g. point
h. line segment

Lesson-by-Lesson Review

1-1 Points, Lines, and Planes

See pages 6–11.

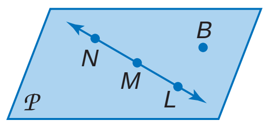
Concept Summary

- A line is determined by two points.
- A plane is determined by three noncollinear points.

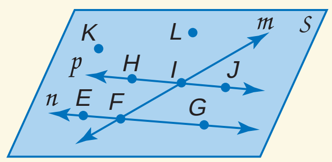
Example **Use the figure to name a plane containing point *N*.**

The plane can be named as plane $\mathcal{P}$.

You can also use any three noncollinear points to name the plane as plane *BNM*, plane *MBL*, or plane *NBL*.



Exercises **Refer to the figure.** *See Example 1 on page 7.*



7. Name a line that contains point *I*.
8. Name a point that is not in lines *n* or *p*.
9. Name the intersection of lines *n* and *m*.
10. Name the plane containing points *E*, *J*, and *L*.

Draw and label a figure for each relationship. *See Example 3 on pages 7–8.*

11. Lines ℓ and m are coplanar and meet at point C.

12. Points S, T, and U are collinear, but points S, T, U, and V are not.

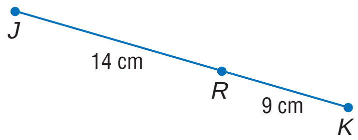
1-2 Linear Measure and Precision

See pages 13–19.

Concept Summary

- The precision of any measurement depends on the smallest unit available on the measuring device.
- The measure of a line segment is the sum of the measures of its parts.

Example **Use the figure to find JK.**



$JK = JR + RK$ Sum of parts = whole

$= 14 + 9$ or 23 Substitution

So, $\overline{JK}$ is 23 centimeters long.

Exercises **Find the value of the variable and PB, if P is between A and B.**
See Example 4 on pages 14 and 15.

13. $AP = 7, PB = 3x, AB = 25$

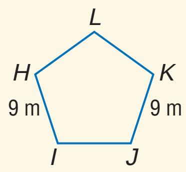
14. $AP = 4c, PB = 2c, AB = 9$

15. $AP = s + 2, PB = 4s, AB = 8s - 7$

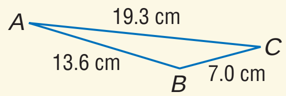
16. $AP = -2k, PB = k + 6, AB = 11$

Determine whether each pair of segments is congruent. *See Example 5 on page 16.*

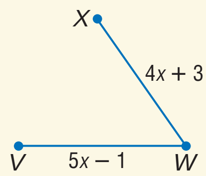
17. $\overline{HI}, \overline{KJ}$



18. $\overline{AB}, \overline{AC}$



19. $\overline{VW}, \overline{WX}$



1-3 Distance and Midpoints

See pages 21–27.

Concept Summary

- Distances can be determined on a number line or the coordinate plane by using the Distance Formulas.
- The midpoint of a segment is the point halfway between the segment's endpoints.

Example **Find the distance between $A(3, -4)$ and $B(-2, -10)$.**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$AB = \sqrt{(-2 - 3)^2 + [-10 - (-4)]^2}$ $(x_1, y_1) = (3, -4)$ and $(x_2, y_2) = (-2, -10)$

$= \sqrt{(-5)^2 + (-6)^2}$ Simplify.

$= \sqrt{61}$ or about 7.8 Simplify.

Exercises **Find the distance between each pair of points.**
See Example 2 on pages 21–22.

20. $A(1, 0)$, $B(-3, 2)$

21. $G(-7, 4)$, $L(3, 3)$

22. $J(0, 0)$, $K(4, -1)$

23. $M(-4, 16)$, $P(-6, 19)$

Find the coordinates of the midpoint of a segment having the given endpoints.
See Example 3 on page 23.

24. $D(0, 0)$, $E(22, -18)$

25. $U(-6, -3)$, $V(12, -7)$

26. $P(2, 5)$, $Q(-1, -1)$

27. $R(3.4, -7.3)$, $S(-2.2, -5.4)$

1-4 Angle Measure

See pages 29–36.

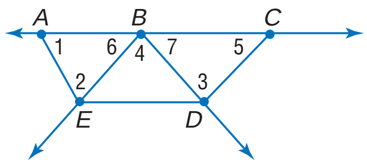
Concept Summary

- Angles are classified as acute, right, or obtuse according to their measure.
- An angle bisector is a ray that divides an angle into two congruent angles.

Examples

a. Name all angles that have *B* as a vertex.

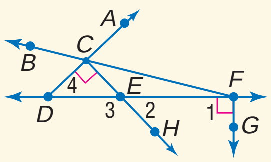
$\angle 6$, $\angle 4$, $\angle 7$, $\angle ABD$, $\angle EBC$



b. Name the sides of $\angle 2$.

$\overline{EA}$ and $\overline{EB}$

Exercises **For Exercises 28–30, refer to the figure at the right.** *See Example 1 on page 30.*



28. Name the vertex of $\angle 4$.

29. Name the sides of $\angle 1$.

30. Write another name for $\angle 3$.

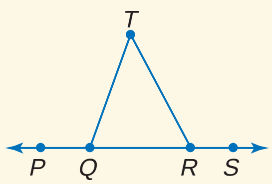
Measure each angle and classify it as *right*, *acute*, or *obtuse*. *See Example 2 on page 30.*

31. $\angle SQT$

32. $\angle PQT$

33. $\angle T$

34. $\angle PRT$

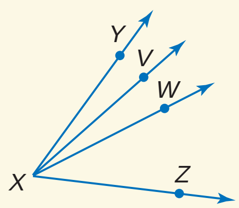


In the figure, $\overrightarrow{XW}$ bisects $\angle YXZ$ and $\overrightarrow{XV}$ bisects $\angle YXW$.
See Example 3 on page 32.

35. If $m\angle YXV = 3x$ and $m\angle VXW = 2x + 6$, find $m\angle YXW$.

36. If $m\angle YXW = 12x - 10$ and $m\angle WXZ = 8(x + 1)$, find $m\angle YXZ$.

37. If $m\angle YXZ = 9x + 17$ and $m\angle WXZ = 7x - 9$, find $m\angle YXW$.



Chapter 1 For More ...

- Extra Practice, see pages 754–755.
- Mixed Problem Solving, see page 782.

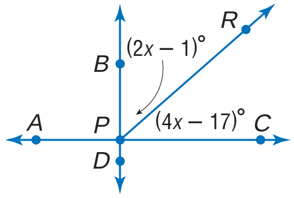
1-5 Angle Relationships

See pages 37–43.

Concept Summary

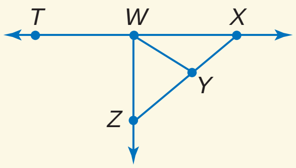
- There are many special pairs of angles, such as adjacent angles, vertical angles, complementary angles, and linear pairs.

Example **Find the value of x so that $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are perpendicular.**



$m\angle BPC = m\angle BPR + m\angle RPC$	Sum of parts = whole
$90 = 2x - 1 + 4x - 17$	Substitution
$108 = 6x$	Simplify.
$18 = x$	Divide each side by 6.

Exercises **For Exercises 38–41, use the figure at the right.**
See Examples 1 and 3 on pages 38 and 40.



38. Name two obtuse angles.

39. Name a linear pair whose angles have vertex W.

40. If $m\angle TWZ = 2c + 36$, find c so that $\overline{TW} \perp \overline{WZ}$.

41. If $m\angle ZWY = 4k - 2$, and $m\angle YWX = 5k + 11$, find k so that $\angle ZWX$ is a right angle.

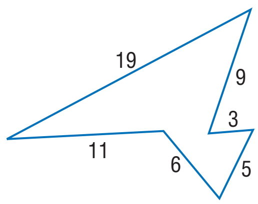
1-6 Polygons

See pages 45–50.

Concept Summary

- A polygon is a closed figure made of line segments.
- The perimeter of a polygon is the sum of the lengths of its sides.

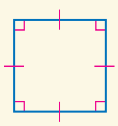
Example **Find the perimeter of the hexagon.**



$P = s_1 + s_2 + s_3 + s_4 + s_5 + s_6$	Definition of perimeter
$= 19 + 9 + 3 + 5 + 6 + 11$ or 53	Substitution

Exercises **Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.** *See Example 1 on page 46.*

42.



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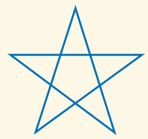
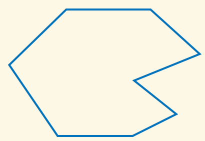


44.



Find the perimeter of each polygon. *See Example 3 on page 47.*

45. hexagon $ABCDEF$ with vertices $A(1, 2)$, $B(5, 1)$, $C(9, 2)$, $D(9, 5)$, $E(5, 6)$, $F(1, 5)$

46. rectangle $WXYZ$ with vertices $W(-3, 5)$, $X(7, 1)$, $Y(5, -4)$, $Z(-5, 0)$

Chapter 1

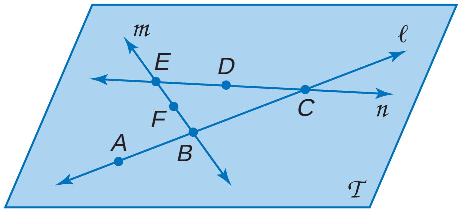
Practice Test

Vocabulary and Concepts

Determine whether each statement is *true* or *false*.

1. A plane contains an infinite number of lines.
2. If two angles are congruent, then their measures are equal.
3. The sum of two complementary angles is 180.
4. Two angles that form a linear pair are supplementary.

Skills and Applications



For Exercises 5–7, refer to the figure at the right.

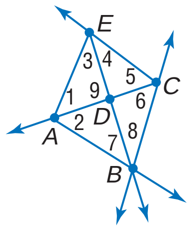
5. Name the line that contains points B and F.
6. Name a point not contained in lines ℓ or m.
7. Name the intersection of lines ℓ and n.

Find the value of the variable and VW if V is between U and W.

8. $UV = 2$, $VW = 3x$, $UW = 29$
9. $UV = r$, $VW = 6r$, $UW = 42$
10. $UV = 4p - 3$, $VW = 5p$, $UW = 15$
11. $UV = 3c + 29$, $VW = -2c - 4$, $UW = -4c$

Find the distance between each pair of points.

12. $G(0, 0)$, $H(-3, 4)$
13. $N(5, 2)$, $K(-2, 8)$
14. $A(-4, -4)$, $W(-2, 2)$



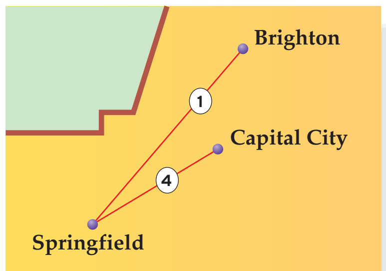
For Exercises 15–18, refer to the figure at the right.

15. Name the vertex of $\angle 6$.
16. Name the sides of $\angle 4$.
17. Write another name for $\angle 7$.
18. Write another name for $\angle ADE$.
19. **ALGEBRA** The measures of two supplementary angles are $4r + 7$ and $r - 2$. Find the measures of the angles.
20. Two angles are complementary. One angle measures 26 degrees more than the other. Find the measures of the angles.

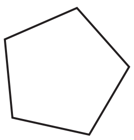
Find the perimeter of each polygon.

21. triangle PQR with vertices $P(-6, -3)$, $Q(1, -1)$, and $R(1, -5)$
22. pentagon $ABCDE$ with vertices $A(-6, 2)$, $B(-4, 7)$, $C(0, 4)$, $D(0, 0)$, and $E(-4, -3)$

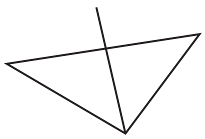
DRIVING **For Exercises 23 and 24, use the following information and the diagram.**
The city of Springfield is 5 miles west and 3 miles south of Capital City, while Brighton is 1 mile east and 4 miles north of Capital City. Highway 1 runs straight between Brighton and Springfield; Highway 4 connects Springfield and Capital City.

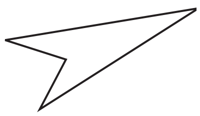


23. Find the length of Highway 1.
24. How long is Highway 4?
25. **STANDARDIZED TEST PRACTICE** Which of the following figures is *not* a polygon?

(A)


(B)


(C)


(D)


Chapter 1 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. During a science experiment, Juanita recorded that she blinked 11 times in one minute. If this is a normal count and Juanita wakes up at 7 A.M. and goes to bed at 10 P.M., how many times will she blink during the time she is awake? (Prerequisite Skill)

Ⓐ 165 Ⓑ 660
Ⓒ 9900 Ⓓ 15,840

2. Find $-\sqrt{0.0225}$. (Prerequisite Skill)

Ⓐ -0.15 Ⓑ -0.015
Ⓒ 0.015 Ⓓ 0.15

3. Simplify $\dfrac{2x^2 + 12x + 16}{2x + 4}$. (Prerequisite Skill)

Ⓐ 24 Ⓑ $x + 4$
Ⓒ $4x + 12$ Ⓓ $4x^3 + x^2 + 20$

4. If two planes intersect, their intersection can be

I. a line.
II. three noncollinear points.
III. two intersecting lines. (Lesson 1-1)

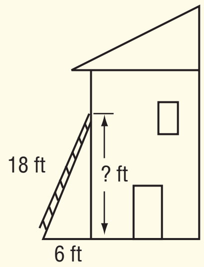
Ⓐ I only Ⓑ II only
Ⓒ III only Ⓓ I and II only

5. Before sonar technology, sailors determined the depth of water using a device called a sounding line. A rope with a lead weight at the end was marked in intervals called fathoms. Each fathom was equal to 6 feet. Suppose a specific ocean location has a depth of 55 fathoms. What would this distance be in yards? (Lesson 1-2)

Ⓐ $9\frac{1}{6}$ yd Ⓑ 110 yd
Ⓒ 165 yd Ⓓ 330 yd

6. An 18-foot ladder leans against the side of a house so that the bottom of the ladder is 6 feet from the house. To the nearest foot, how far up the side of the house does the top of the ladder reach? (Lesson 1-3)

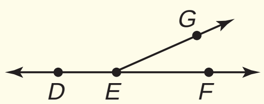
Ⓐ 12 ft
Ⓑ 14 ft
Ⓒ 17 ft
Ⓓ 19 ft



7. Ray BD is the bisector of $\angle ABC$. If $m\angle ABD = 2x + 14$ and $m\angle CBD = 5x - 10$, what is the measure of $\angle ABD$? (Lesson 1-5)

Ⓐ 8 Ⓑ 16
Ⓒ 30 Ⓓ 40

8. If $m\angle DEG$ is $6\frac{1}{2}$ times $m\angle FEG$, what is $m\angle DEG$? (Lesson 1-6)



Ⓐ 24 Ⓑ 78
Ⓒ 130 Ⓓ 156

9. Kaitlin and Henry are participating in a treasure hunt. They are on the same straight path, walking toward each other. When Kaitlin reaches the Big Oak, she will turn 115° onto another path that leads to the treasure. At what angle will Henry turn when he reaches the Big Oak to continue on to the treasure? (Lesson 1-6)

Ⓐ 25° Ⓑ 35°
Ⓒ 55° Ⓓ 65°

Aligned and verified by The Princeton Review

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

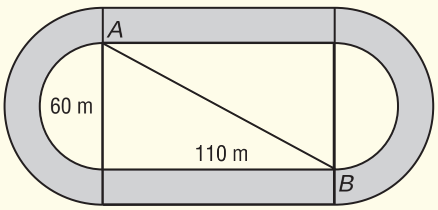
10. Simplify $-2x + 6 + 4x^2 + x + x^2 - 5$. (Prerequisite Skill)

11. Solve the system of equations. (Prerequisite Skill)
$2y = 3x + 8$
$y = 2x + 3$

12. In rectangle $ABCD$, vertices A, B, and C have the coordinates $(-4, -1)$, $(-4, 4)$, and $(3, 4)$, respectively. Plot A, B, and C and find the coordinates of vertex D. (Lesson 1-1)

13. The endpoints of a line segment are $(2, -1)$ and $(-4, 3)$. What are the coordinates of its midpoint? (Lesson 1-3)

14. The 200-meter race starts at point A, loops around the track, and finishes at point B. The track coach starts his stopwatch when the runners begin at point A and crosses the interior of the track so he can be at point B to time the runners as they cross the finish line. To the nearest meter, how long is $\overline{AB}$? (Lesson 1-3)



15. Mr. Lopez wants to cover the walls of his unfinished basement with pieces of plasterboard that are 8 feet high, 4 feet wide, and $\frac{1}{4}$ inch thick. If the basement measures 24 feet wide, 16 feet long, and 8 feet tall, how many pieces of plasterboard will he need to cover all four walls? (Lesson 1-4)

The Princeton Review Test-Taking Tip

Question 8 Most standardized tests allow you to write in the test booklet or on scrap paper. To avoid careless errors, work out your answers on paper rather than in your head. For example, in Question 8, you can write an equation, such as $x + 6\frac{1}{2}x = 180$ and solve for x. All of your solution can be examined for accuracy when written down.

Part 3 Open Ended

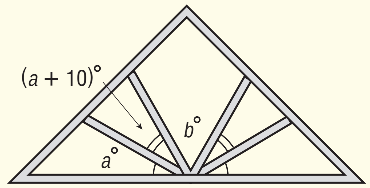
Record your answers on a sheet of paper. Show your work.

16. Tami is creating a sun catcher to hang in her bedroom window. She makes her design on grid paper so that she can etch the glass appropriately before painting it.

a. Graph the vertices of the glass if they are located at $(4, 0)$, $(-4, 0)$, $(0, -4)$, and $(0, 4)$. (Prerequisite Skill)

b. Tami is putting a circle on the glass so that it touches the edge at the midpoint of each side. Find the coordinates of these midpoints. (Lesson 1-3)

17. William Sparrow and his father are rebuilding the roof of their barn. They first build a system of rafters to support the roof. The angles formed by each beam are shown in the diagram.



a. If $a = 25$, what is the measure of the five angles formed by the beams? Justify your answer. (Lesson 1-6)

b. Classify each of the angles formed by the beams. (Lesson 1-5)